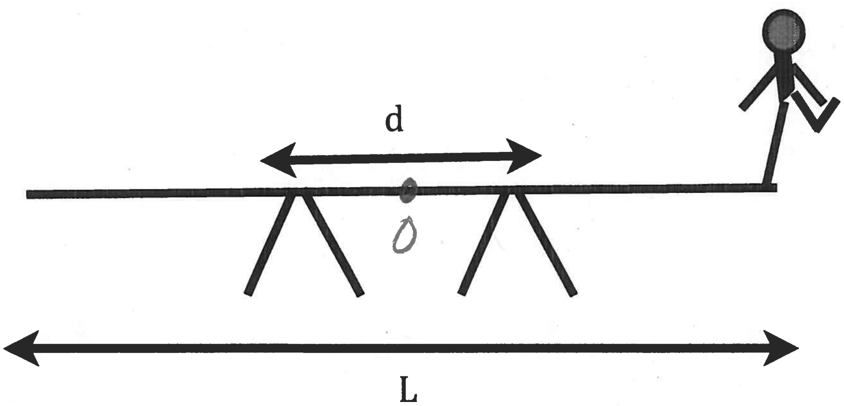
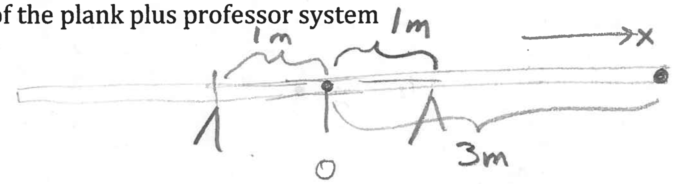
Name: *Solutions*

1. [25 pts] A uniform wooden plank of length L=6.0 m and mass M=90 kg rests of top of two sawhorses separated by d= 2.0 m and located equal distances from the center of the plank as shown in the picture. A physics professor stands on the right-hand end of the plank. *You must show your work to receive credit.*



a) [10 pts] Let m be the mass of the professor. Write down an expression for the center of mass position of the plank plus professor system



$$COM_x = \frac{m_{plank} r_{pl} + m_{prof} r_{prof}}{m_{pl} + m_{prof}}$$

$$COM_x = \frac{0 + (L/2) m}{m + M}$$

$$\boxed{COM_x = \frac{\frac{L}{2} m}{m+M}, \text{ if center is origin}}$$

b) [8 pts] What is the maximum mass the professor can have and still ensure that the plank remains in equilibrium?

need $COM_x \leq 1\,m \; (\frac{d}{2})$ (where sawhorse is), so

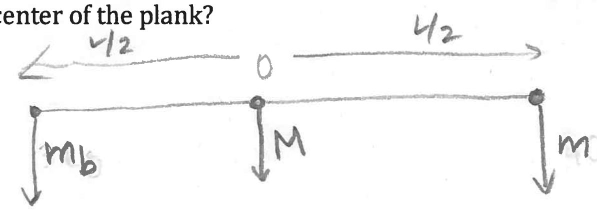
$$COM_x = \frac{d}{2} = \frac{L/2\, m}{(m+M)} \qquad d = \frac{Lm}{m+M} \Rightarrow dm + dM = Lm$$

$$dM = m(L-d)$$

$$m = \frac{dM}{(L-d)} = \frac{180.}{(6-2}$$

$$\boxed{m_{max} = 4}$$

c) [7 pts] Assume the professor weighs 40 kg. A bucket containing water with a total mass of 3 kg is placed at the left-hand end of the plank. Treat the professor and the bucket as point masses. What is now the distance of the center of mass of the combined system (professor+plank+bucket) from the center of the plank?



$$COM_x = \frac{-m_b L/2 + m L/2}{m_b + m + M}$$

$$= \frac{3m\,(m - m_b)}{m_b + m + M} = \frac{37\,kg\,(3)\,m}{(43+90)\,kg}$$

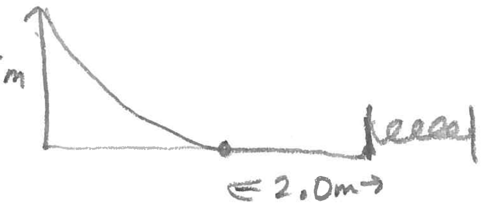
$$= \frac{111}{133}\,m$$

$$\boxed{COM_x = 0.83\,m}$$

Name: solutions

2. [25 pts] Starting from rest, a 5.0 kg package slides down a 5.0 m high frictionless ramp, across a 2.0 m wide horizontal surface, and then hits a spring with a spring constant of 500 N/m. The other end of the spring is anchored against a wall. The ground under the spring is frictionless, but the 2.0 m wide horizontal surface is rough, with a coefficient of kinetic friction of 0.25 with the box. *You must show your work to receive credit.*

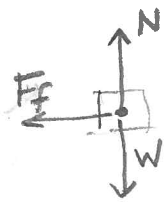
a) [5 pts] What is the speed of the package just before reaching the rough surface?



$mgh = \frac{1}{2}mv^2 \quad v = \sqrt{2gh}$

$v = \sqrt{2(9.8\ m/s^2)\,5m} \quad \boxed{v = 9.89\ m/s}$

b) [5 pts] Draw a free body diagram for the package as it moves across the rough horizontal surface, and use it to calculate the net work done on the package as it slides across the 2.0 m wide horizontal surface.



$N = W \Rightarrow N = mg$

$F_f = \mu_K N = \mu_K mg$

$W = -\vec{F_f} \cdot \vec{d}$

$W = -\mu_K mgd = -(0.25)(5\ kg)(9.8\ m/s^2)(2.0\ m) = -24.5\ J$

c) [5 pts] What is the speed of the package just before hitting the spring?

$\Delta K = W_{noncons} \Rightarrow KE_f - KE_i = W_{noncons}$

$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - W \Rightarrow v_f^2 = v_i^2 - \frac{2W}{m} =$

$98(m/s)^2 - 9.8(m/s)^2 = 88.2$

$\boxed{v_f = 9.39\ m/s}$

d) [5 pts] How far is the spring compressed?

$\frac{1}{2}mv_f^2 = \frac{1}{2}kx^2 \qquad x^2 = \frac{mv_f^2}{k} = \frac{(5\ kg)\ 88.2(m/s)^2}{500\ N/m} = 0.88\ m$

$\boxed{x = 0.93\ m}$

e) [5 pts] Including the first crossing, how many complete trips will the box make across the rough surface before coming to rest?

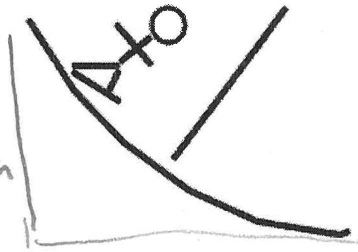
$KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(5\ kg)(9.89\ m/s)^2 = 245\ J$

$W_{noncons} = N(+24.5\ J) = 245\ J$

$N = 10$ trips

Name: ______________________________

3. (25 pts total) A student skateboards down a frictionless playground ramp in the shape of a quarter circle with radius R=3.0 m.The total mass of the student and his skateboard is 25 kg.

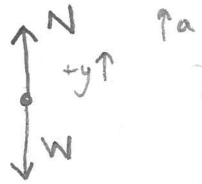


a) (5 pts) What is speed of the student at the bottom of the ramp (assume g=9.8 m/s^2) ?

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \qquad v = \sqrt{2gh} = \sqrt{2(9.8)(3)\ \text{m/s}^2\cdot\text{m}}$

$v = 7.6\ \text{m/s}$

b) (5pts) Calculate the normal force that acts on the student at the bottom of the ramp. What is its direction ?



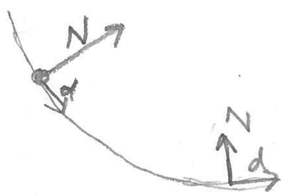
$N - W = \frac{mv^2}{R} \Rightarrow N = mg + \frac{mv^2}{R}$

$= 25\,\text{kg}\left(g + \frac{v^2}{R}\right) = 25\,\text{kg}(9.8 + 19.6)$

$\boxed{N = 735\ \text{N}}$

c) (5pts) How much work does the normal force do as the student descends the ramp ?

$W = \vec{F}\cdot\vec{d} = \vec{N}\cdot\vec{d} = 0$



d) (5pts) At the same time that skateboarder starts down the ramp a second student jumps from the top of the ramp and falls vertically to the ground. With what speed will this second student hit the ground ?

$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \qquad v = \sqrt{2gh} = 7.6\ \text{m/s}$ or $v_f^2 = v_i^2 + 2a\Delta x$

$\Rightarrow v = \sqrt{2gh}$

e) (5 pts) Which student will reach ground level first ? Give a reasoned argument.

The student who jumps straight down lands first. While the magnitudes of the velocity vectors are the same at every height, the component of the velocity vector in the vertical direction is always larger for the "jumping/falling" student than the skateboarder. Therefore, the amount of time requiried to displace the same vertical distance is shorter for the jumper. This is very similar to the demo in class with the 4 balls that take diffrent paths to reach the end of a ramp.

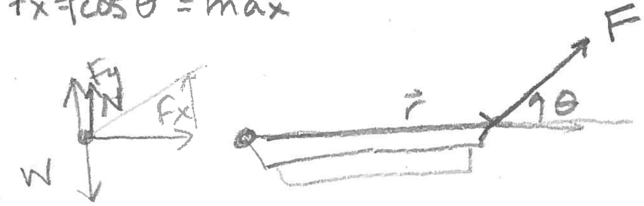
Name: ______________________________

4. [25 pts] A girl pulls her 15-kg sled along a flat frozen pond by applying a 10-N force to the front edge of the sled at 37° to the horizontal. Assume that friction is negligible and that the sled starts from rest.

(a) [6 pts] How much work does the girl do on the sled in the first 2.0 s?

$a_x =$

$F_x = F\cos\theta = ma_x$



$W = \vec{F} \cdot \vec{d}$

$d = \frac{1}{2} a_x t^2$

$W = Fd\cos\theta$

$d = \frac{1}{2} \frac{F\cos\theta}{m} t^2$

$W = \frac{F^2 \cos^2\theta}{2\,m} t^2 = \frac{N^2}{m}(2.12)(2\,s) = 4.25\ J$

b) [4 pts] What is the torque generated on the sled by the girl? Assume the pivot point is the back edge of the sled, which is 1 m long.

$\tau_F = \vec{r} \times \vec{F} = (1m)(10N)\sin\theta = 6.0\ N \cdot m$

(c) [5pts] How much instantaneous power does she exert at *t*=2.0s?

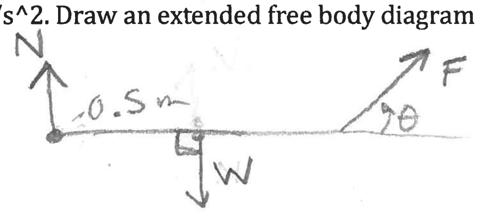
$V_f = \cancel{V_i}^{\,0} + a_x t$ $\quad a_x = \frac{F\cos\theta}{m}$

$P = \vec{F} \cdot \vec{V}$

$V_f = \frac{F\cos\theta}{m} t = 1.06\ m/s$

$P = \frac{F^2\cos^2\theta}{m} t = F V_f \cos\theta = 8.46\ J/s$

d) [4 pts] Assume the girl still pulls the sled at the same angle from the center of the front edge, but now with a force F, that is large enough generate an initial instantaneous angular acceleration of α = 1 rad/s^2. Draw an extended free body diagram for this situation.



e) [6 pts] Assuming the pivot point is at the back edge of the sled, and that the sled can be modeled as a flat plate with length 1m (front-to-back) and 0.5m (side-to-side), what is the initial magnitude of force F with which the girl must be pulling?

$\tau_W = -mg\,L/2 = 73.5\ N \cdot m$ $\quad \tau_{NET} = I\alpha$ $\quad I = \frac{1}{3} ML^2 = \frac{1}{3}(15\,kg)(1\,m)^2 = 5\ kg \cdot m^2$

$\tau_F = LF\sin\theta$

$\tau_N = 0$

$\tau_{NET} = LF\sin\theta - mg\,L/2 = \underbrace{I\alpha}_{5.0\,kg \cdot m^2/s^2}$

$F = \frac{78.5\ N \cdot m}{L\sin\theta} = 130\,N$