Homework (Week-12) Solutions!

Ch - 10

Problem 59:

$V_x = 40.0 \, m/s.$

time taken to drop a height of 300 m.

$$t = \sqrt{2gh} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(300)}{10}} = \sqrt{60}$$
$$= 7.74 \text{ sec}$$

$$V_y = 0 + (-g)t$$
$$= (-10)(7.74) = -77.45 \, m/s.$$

(a) $V = \sqrt{V_x^2 + V_y^2} = 87.16 \, m/s.$

~~(b)~~ $\cancel{V_{cm} = V = \omega r}$. $\cancel{\omega = \frac{V_{cm}}{r}} = \cancel{87.16}$

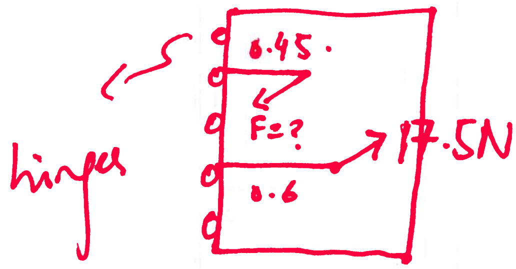
(b) $mgh = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \omega^2.$

$V_{cm} \neq \omega r.$ (for rolling without slipping)

but we can't use this in this case since there is no rolling

$V_{cm} = 87.16 \, m/s.$

Problem 117:



Counter Clockwise torque = $(0.6)(17.5)$.

Clock-wise torque = $(F)(0.45)$

to keep the door from moving:

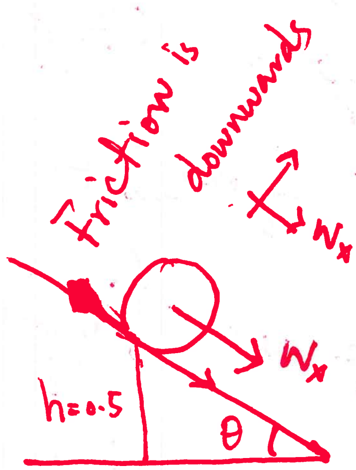
$$\tau_{net} = 0.$$

$$\Rightarrow \quad F(0.45) = (0.6)(17.5)$$

$$F = \frac{0.6}{0.45}(17.5) = 23.33N.$$

Ch- 11

Problem 28:



$W_x = mg\sin\theta$.

$mg\sin\theta + F_f = ma_{cm}$

for linear motion

In this case we have rolling friction

$$\tau_{net} = I\alpha$$

$$F_f \cdot R = I\alpha$$

Work done by friction in this case is zero. Work done by friction is negative when we consider translational (linear) motion, while it is positive for rotational motion since friction creates torque and do positive work done. Adding the work done by friction for both motions, we have net zero work done by friction if the object is rolling without slipping. Another explanation is that the point of contact of the bouncing ball has net zero displacement under the impact of friction, so work done by rolling friction is zero.

Using conservation of energy we have:

$$E_i = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$ (at the bottom of the ramp)

$$\omega = \frac{v_{cm}}{R}.$$

$$\Rightarrow E_i = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} \frac{\frac{2}{5} m \cancel{R^2}}{} \frac{v_{cm}^2}{\cancel{R^2}}$$

$$= \left(\frac{1}{2} + \frac{1}{5}\right) m v_{cm}^2$$

$$E_f = \left(\frac{1}{2} + \frac{1}{5}\right) m \left(v_{cm}^{final}\right)^2 + mgh.$$

$$E_i = E_f.$$

$$\frac{7}{10} m v_{cm}^2 = \frac{7}{10} m \left(v_{cm}^{final}\right)^2 + mgh.$$

Problem 36:

Treating the race-car as a point mass

$$L = mvr.$$

$$L_1 = m v_1 r_1 \qquad L_2 = m v_2 r_2.$$

$$\frac{L_2}{L_1} = \frac{m v_2 r_2}{m v_1 r_1} = \frac{v_2 r_2}{v_1 r_1}$$

Problem 44:

$L^{orbit} = mvr.$ for orbit around the sun.

$L^{spin} = \bar{I}\omega$ for spinning about it's own-axis.

$$\frac{L^{orbit}}{L^{spin}} = \frac{mvr}{\frac{2}{5} mR^2 \omega}.$$

$V \equiv$ orbital velocity

ω = angular velocity.

$r \equiv$ distance from centre of Earth to centre of Sun.

$R \equiv$ Radius of Earth.

Problem 58:

Angular momentum of particle about the centre of cylinder right before collision.

$L = mvr = \vec{r} \times \vec{p}.$

$L^{particle} = \frac{20}{1000} \cdot (10)(0.1).$

$L_i = L^{particle} + (L_{cylinder} = 0)$

$L_f = I\omega.$ where $I = \frac{1}{2} mr^2 + m_{particle} r^2.$

$$\omega = \frac{(20 \times 10^{-3})(10)(0.1)}{\frac{1}{2}(0.5)(0.1)^2 + (20 \times 10^{-3})(0.1)^2}$$ using $L_i = L_f$

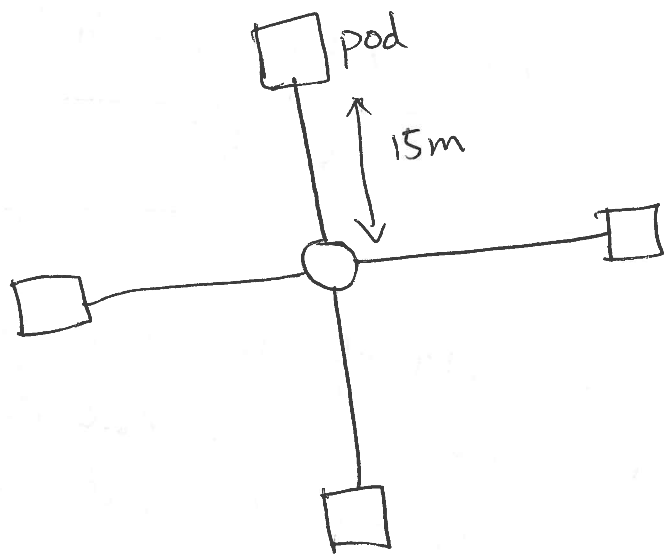
$$K_i = \frac{1}{2} m_{particle} V^2 = \frac{1}{\cancel{2}} (\overset{10}{\cancel{20}} \times 10^{-3})(10)^2$$

$$= 10^{-2} \cdot 10^2 = 1\,J.$$

$$K_f = \frac{1}{2} I \omega^2$$

$$\text{Energy loss} = K_i - K_f.$$

Problem 71:



$$I_{ride} = 4\, I_{spoke} + 4\, I_{pod+children}$$

$$I_{ride} = 4 \times \left(\frac{1}{3}(200)(15)^2\right) + \left((200)(15)^2 \times 4\right).$$

$$= \frac{4}{3}(200)(15)^2 + (800)(15)^2 = \left(800 + \frac{4}{3} \cdot 200\right)(15)^2$$

$$L^{initial}_{ride} = I_{ride}\, \omega_{initial} = (2.4 \times 10^5)(0.2 \times 2\pi)$$

After children jump off.

$$I_{ride} = 4I_{spoke} + 4I_{pods}.$$

$$= 4 \cdot \frac{1}{3}(200)(15)^2 + 4(100)(15)^2.$$

$$= 1.5 \times 10^5.$$

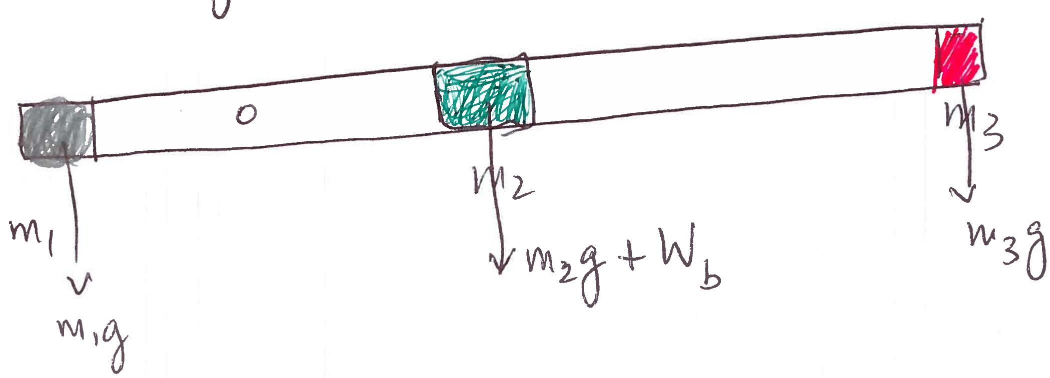
$$L_{ride}^{final} = (1.5 \times 10^5)(\omega'_{final}).$$

$$\omega'_{final} = \frac{(2.4 \times 10^5)(0.2 \times 2\pi)}{1.5 \times 10^5}.$$

Additional problem:

(a) $2r$. is where the COM of the beam is; but since the system is balanced the COM of the system is right at the hinge.

(b)



(c) $m_1 g \cancel{r} = (m_2 g + W_b) 2\cancel{r} + m_3 g(5\cancel{r})$.

$$m_1 g = (m_2 g + W_b) 2 + m_3 g.$$

$$m_1 \cancel{g} = (m_2 \cancel{g} + 0.5\cancel{g}) 2 + 5 m_3 \cancel{g}$$

$$2.5 = (0.5 + 0.5)(2) + 5m_3.$$

$$2.5 - 2 = 5m_3.$$

$$m_3 = \frac{0.5}{5} kg.$$

$$m_3 = 0.1 kg$$

(d) $N = (0.5 + 2.5 + 0.5 + 0.1) g.$

$N = 3.6g.$

(e) $\tau = (10 \sin 60^\circ)(5r).$

(f) $I_{hinge} = \frac{1}{12} m l^2 + m(2r)^2 + m_1 (r)^2 + m_2 (2r)^2 + m_3 (5r)^2.$

$= \frac{1}{12}(0.5)(6r)^2 + (0.5)(2r)^2 + 2.5r^2 + 0.5(2r)^2 + (0.1)(5r)^2.$

(g) $\tau = I\alpha$

$$\Rightarrow \quad \alpha = \frac{(10 \sin 60^\circ)(5r)}{\underline{I}_{hinge}}.$$

(h) $\omega_f = \omega_i + \alpha t.$

$\omega_f = 0 + (\alpha)(0.5).$

(i) $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2.$

$= 0 + \frac{1}{2}(\alpha)(0.5)^2$