Homework (Week - 15) Solutions!

Ch - 13

Problem 15:

(a) $$F_g = \frac{G m_b m_f}{r^2} = \frac{6.7 \times 10^{-11} \times 4.20 \times 100}{(0.2)^2}$$

$$= 7.0349 \times 10^{-7}\ N.$$

(b) $$F_g = \frac{G m_b m_J}{r^2} = \frac{(6.7 \times 10^{-11})(4.20)(1.898 \times 10^{27})}{(6.29 \times 10^{11})^2}$$

$$= 1.349 \times 10^{-6}\ N.$$

Gravitational forces from baby's father and other objects added up would be roughly equal to force exerted on the baby by jupiter.

Problem 20:

(a) $g = \frac{GM_e}{r^2}$

$$M_e = \frac{gr^2}{G} = \frac{9.832 \times (6.356 \times 10^6)^2}{6.7 \times 10^{-11}}$$

$$= 5.928 \times 10^{24}$$

M_e from NASA fact sheet is within 0.74% error of the mass calculated above.

Problem 32:

$$U_i = -\frac{Gm_1 m_2}{r_i} = \frac{(6.7 \times 10^{-11})(25)}{0.15}$$

$$= -1.116 \times 10^{-8}\ J.$$

Upon impact their centre-to-centre distance will be 10.2 cm.

$K_i = 0$ $\quad K_f = \left(\frac{1}{2} mv^2\right)(2)$

Upon impact $\rightarrow$ They both will have the same velocity magnitude

$$K_f + U_f = K_i + U_i$$

$r_i = 0.15\,m$.

$r_f = 0.102\,m$.

$$mv^2 = U_i - U_f.$$

$m_1 = m_2 = m$.

$$v^2 = -\frac{Gm_1m_2}{r_i} + \frac{Gm_1m_2}{r_f}$$

Problem 40:

Using Kepler's third law.

$$T^2 = \frac{4\pi^2}{GM_s} r^3.$$

$$M_s = \frac{4\pi^2}{GT^2} r^3$$

$r = 1\,AU$.

Problem 46:

$$T = 1\,\text{day} = (24 \times 3600)\,\text{sec}$$

$$T^2 = \frac{4\pi^2}{GM_E} r_{orbit}^3$$

$$r_{orbit} = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}}$$

Problem 64:

(a) Earth's rotation should be taken into account for determining the real weight.

At the pole; Earth's rotation doesn't affect your weight. In other word's it is your true weight due to Earth's gravitational pull. At the equator using the net centripetal force experienced by a mass, your weight is

$$mg - \frac{mv^2}{r} = W \Rightarrow W = m\left(g - \frac{v^2}{r}\right) = m(g - \omega^2 r)$$

$$W_{equator} = m(g - \omega^2 R_E)$$ where

$r = R_E$ at the equator.

$r = 0$ at the pole.

$$W_{pole} = mg.$$

$$W_{equator} = \frac{1}{2} W_{pole}.$$

$$\cancel{m}(g - \omega^2 R_{planet}) = \frac{1}{2}\cancel{m}g.$$

$$g - \frac{1}{2}g = \omega^2 R_E.$$

$$\omega^2 = \frac{\frac{1}{2}g}{R_E} = \frac{\frac{1}{2}(9.81)}{6.4 \times 10^6}$$

$$(T^2)^{-1} = \frac{(4\pi^2)^{-1}(\frac{1}{2}g)}{R_E}$$

$$(T^2)^{-1} = \frac{(2\pi^2)^{-1} g \cdot \frac{1}{2}}{R_E}.$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{\frac{1}{2}g}{R_E}$$

$$\frac{2(4\pi^2)R_E}{g} = T^2.$$

you don't need to consider the shape.

Ch-15:

Problem 21:

$$x(t) = A\sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -A\omega^2 \sin(\omega t + \phi)$$

$$\ddot{x} = -\omega^2 x$$

$$a = -\omega^2 x \quad (SHM)$$

Cosine function is chosen since we have the mass and spring having an initial displacement which maximum. at $t = 0$.

$$x(t) = A \cos(\omega t + \phi)$$

$$\Rightarrow \quad \frac{d^2 x}{dt^2} = -\omega^2 x$$ same period of oscillations.

Problem 27:

Each piston has a sound frequency of $\frac{750}{8}$.

so, after in two revolutions. piston makes a sound of frequency $\frac{750}{8} = 93.75$ Hz.

$$\omega = (93.75)(2) \text{ rev/s}$$

Speed of the car is $V = (93.75)(2)\left(\frac{1000}{2000}\right)$

$= \omega r$

$= 93.75$ m/s.

Problem 30: Use $T = 2\pi \sqrt{m/k}$

$m = 0.5$, $T = 1.50 \Rightarrow k = \frac{4\pi^2 m}{(1.50)^2}$

Now if $T = 2$ sec

$$2 = 2\pi \sqrt{\frac{m'}{4\pi^2 m/(1.50)^2}}$$

$$= 2\pi \sqrt{\frac{m'(1.50)^2}{4\pi^2 m}}$$

$$2 = \frac{\cancel{2\pi}}{\cancel{2\pi}} (1.50) \sqrt{\frac{m'}{m}}$$

$$\left(\frac{4}{3}\right)^2 = \frac{m'}{m}$$

$$m' = \frac{16}{9} m$$

$$m' = \frac{16}{9}(0.5) = 0.88 \text{ kg}.$$

We need to add a mass of 0.38 kg to make the period equal to 2 secs.

(d) $F_c = \frac{GMm}{r^2}$

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM_{asteriod}}{r}} = \sqrt{\frac{(6.67\times10^{-11})(1.1\times10^{16})}{(8.75+5.40)\times10^3}}$$

(e) $$V_{esc} = \sqrt{\frac{2GM_{asteriod}}{r_{asteriod}}}$$

$$= \sqrt{\frac{2(6.67\times10^{-11})(1.1\times10^{16})}{(8.75\times10^3)}}$$

Additional Problem:

(a) $W_{mar} = m g_{mars}$ $\qquad g_{mars} = 3.72076 \ m/s^2$

(b) $E = \frac{1}{2}mv^2 + U_g$. where $U_g = 0$ for $r \to \infty$.

$$U_g = -\frac{GMm}{R_{mars}}$$

$$E_i = \frac{1}{2}mv_i^2 + \frac{GMm}{R_{mars}}$$

(c) For escaping. we assume $E_f = \frac{1}{2}mv_{esc}^2 + U_g^f$.

$$U_g = 0$$

$$E_i = E_f$$

$$\frac{1}{2}\cancel{m}v_{esc}^2 = \frac{GM\cancel{m}}{R_{mars}}$$

$$v_{esc} = \sqrt{\frac{2GM}{R_{mars}}}$$