Welcome back to Physics 215

Today's agenda:

- *Oscillators*
  - *SHO*
  - *physical pendulum*
  - *Forced harmonic oscillator*
- *Introduction to waves*

Left to do:

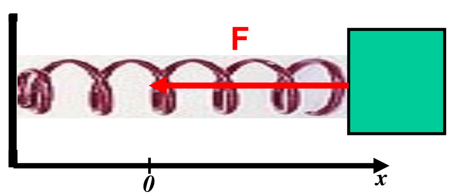
- Evaluations:
  - Please fill out the regular course evaluations online
  - Please fill out the honors evaluation forms at the end of class today
- HW:
  - Problem Set Week 15 Optional (but content for final exam)
  - Post assessment mechanical survey on blackboard (30 minutes, 1 HW grade)
- Final Exam:
  - This will be 3-5pm on Thursday, Dec 12, in Physics 208
  - Same format as other exams, except 6 problems instead of 4.
  - Comprehensive. Covers everything we learned in class (except waves today)

Thanks for being such a great class!

Simple Harmonic Oscillator

$F_x = -k\,x$

Spring constant



Newton's 2nd Law for the block: $m\,a_x = F_x$

Differential equation for x(t)

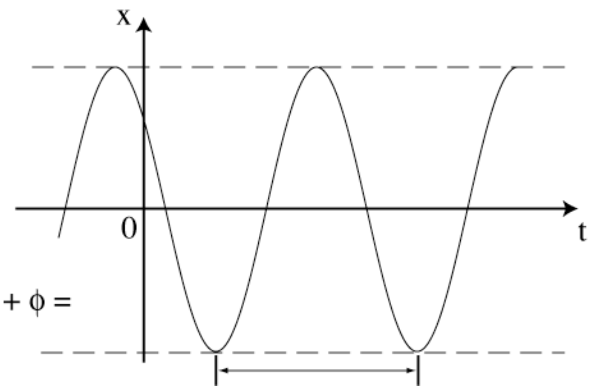
$$m\frac{d}{dt}\left(\frac{dx}{dt}\right) = -\,k\,x$$

Simple Harmonic Oscillator

$$x(t) = A\cos(\omega t + \phi)$$

initial phase

amplitude **angular frequency**



$\omega t + \phi =$

Units:
A - *m*
T - *s*
f - 1/s = Hz (Hertz)
ω - rad/s

f – frequency
Number of oscillations per unit time

$$f = \frac{1}{T} \qquad T = \frac{2\pi}{\omega} \qquad \omega = \frac{2\pi}{T} = 2\pi f$$

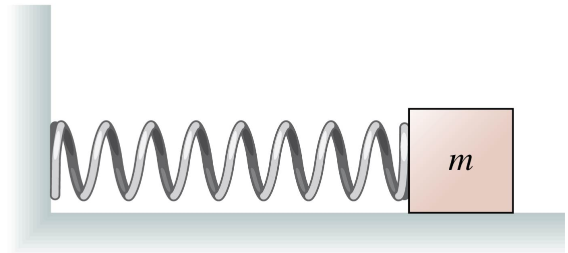
T – *Period*
Time taken by one full oscillation

Simple Harmonic Oscillator DEMO

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

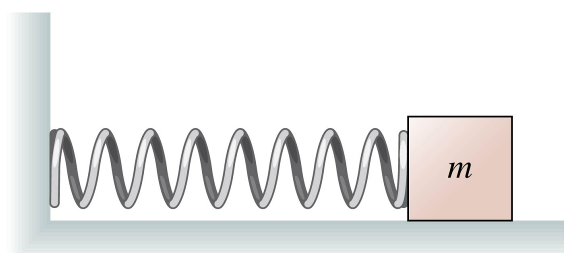
SG A mass oscillates on a horizontal spring with period $T = 2.0$ s. If the amplitude of the oscillation is doubled, the new period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s.



SG A block of mass m oscillates on a horizontal spring with period $T = 2.0$ s. If a second identical block is glued to the top of the first block, the new period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s.



Simple Harmonic Oscillator Total Energy

$$E = U + K$$

$$E = \frac{1}{2}\,k\,x^2 + \frac{1}{2}\,m\,v^2 \qquad v = \frac{dx}{dt}$$

$$E = \frac{1}{2}k\,[A\cos(\omega t + \phi)]^2 + \frac{1}{2}m\,[-A\omega\sin(\omega t + \phi)]^2$$

E =

Simple Harmonic Oscillator -- Summary

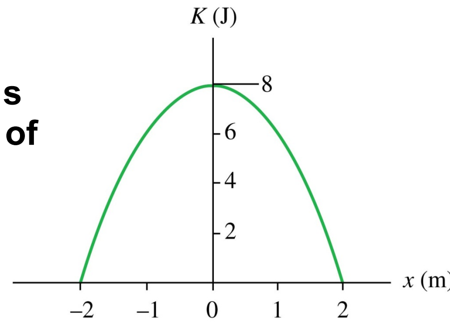
If $\mathbf{F = -k\,\mathit{x}}$ ***then***

$$x(t) = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{T}$$

$$E = \frac{1}{2}\,k\,A^2$$

SG A block oscillates on a very long horizontal spring. The graph shows the block's kinetic energy as a function of position. What is the spring constant?

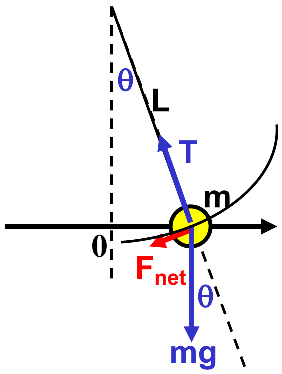


- **A. 1 N/m.**
- **B. 2 N/m.**
- **C. 4 N/m.**
- **D. 8 N/m.**

Importance of Simple Harmonic Oscillations

- For all systems near stable equilibrium
  - $F_{net} \sim -x$ where x is a measure of small deviations from the equilibrium
  - All systems exhibit harmonic oscillations near the stable equilibria for small deviations
- Any oscillation can be represented as superposition (sum) of simple harmonic oscillations (via Fourier transformation)
- Many non-mechanical systems exhibit harmonic oscillations (e.g., electronics)

(Gravitational) Pendulum
Simple Pendulum – Point-like Object



What is the force tangent to the curve?

$F_{net} = -mg \sin\theta$

For small θ (in radians) we can use the small angle approximation:

$F_t = - mg\ s/L$

"Pointlike" – size of the object small compared to L

$T = \frac{2\pi}{\omega} =$

SG Two pendula are created with the same length string. One pendulum has a bowling ball attached to the end, while the other has a billiard ball attached. The natural frequency of the billiard ball pendulum is:

A. greater

B. smaller

C. the same

as the natural frequency of the bowling ball pendulum.

SG The bowling ball and billiard ball pendula from the previous slide are now adjusted so that the length of the string on the billiard ball pendulum is shorter than that on the bowling ball pendulum. The natural frequency of the billiard ball pendulum is:

A. greater

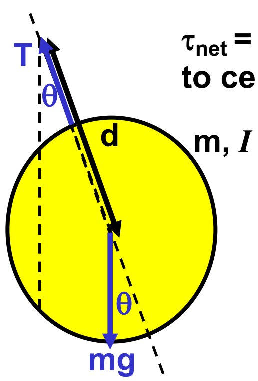
B. smaller

C. the same

as the natural frequency of the bowling ball pendulum.

(Gravitational) Pendulum

Physical Pendulum – Extended Object



τ_{net} = -d mg sinθ, d is distance from pivot to center of mass

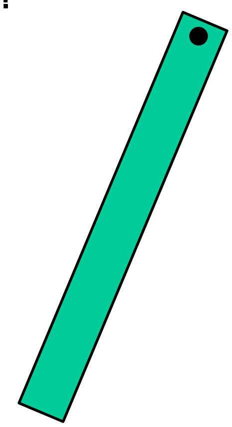
For small θ: sinθ ≈θ

τ_{net} = -d mg θ

$$T = \frac{2\pi}{\omega} =$$

DEMO

Sample problem: A physical pendulum consists of a uniform rod of length 0.90 m and mass M = 5 kg, hinged at one end. What is the period of its oscillations?



Torsion Pendulum
(Angular Simple Harmonic Oscillator)

$\tau = -\kappa\,\theta$

Torsion constant

$$I\alpha = \tau$$

$$I\,\frac{d}{dt}\left(\frac{d\theta}{dt}\right) = -\,\kappa\,\theta$$

Solution: $\theta(t) = A\cos(\omega t + \phi)$

Forced Harmonic Oscillator

Differential equation for x(t): $$m\frac{d}{dt}\left(\frac{dx}{dt}\right) = -kx - b\frac{dx}{dt} - F_0\cos(\omega_d t)$$

Damping force **Driving force**

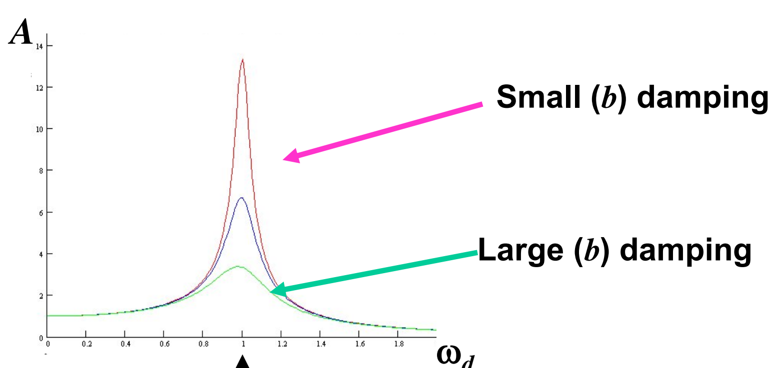
Natural frequency $\omega_0 = \sqrt{\dfrac{k}{m}}$

Solution yields amplitude of response:

$$A = \frac{F_0}{\sqrt{(k - m\omega_d^2)^2 + (b\omega_d)^2}}$$

Resonance

$$A = \frac{F_0}{\sqrt{(k - m\omega_d^2)^2 + (b\omega_d)^2}}$$



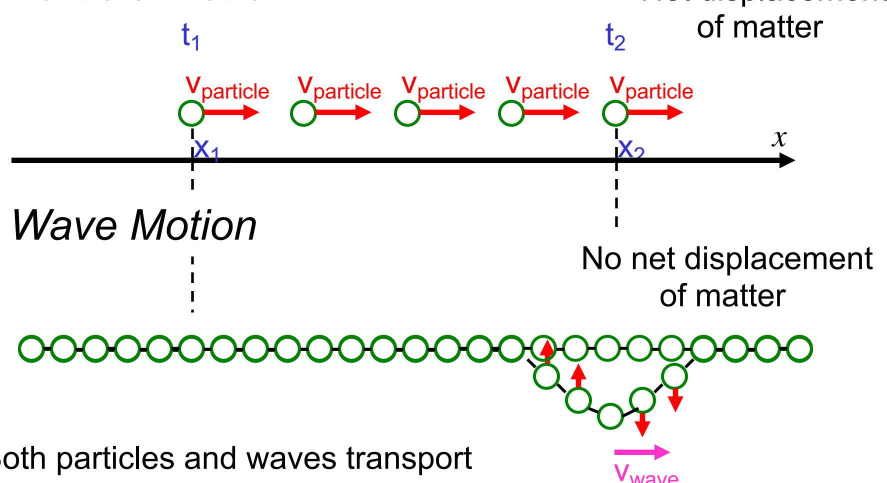
$\omega_d = \omega_0$

Waves – *general features*

- many examples:
  - water waves, musical sounds, seismic tremors, light, gravity, etc.
- system disturbed from equilibrium can give rise to a disturbance which propagates through medium (carries energy)
- periodic character both in time and space
- interference

Wave Motion *vs.* Particle Motion

- *Particle Motion*
- *Wave Motion*



Both particles and waves transport energy from one place to the other

Demo -- waves on rods

Notice:

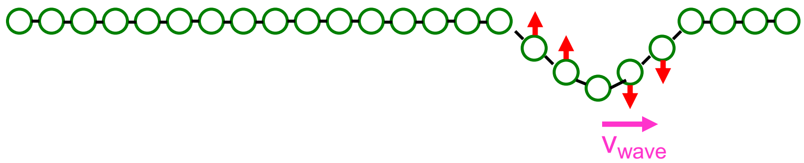
- mean position of rods does not change.
  - but energy transported!
- each rod undergoes periodic motion
- speed of wave does not depend on how fast or what magnitude of driving force
- example of traveling periodic wave

Waves – *structure*

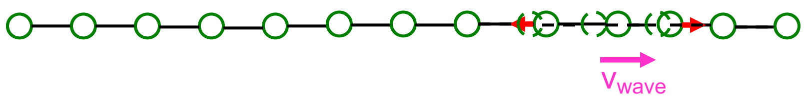
- Wave propagates by making particles in medium execute ***simple harmonic motion***
  - motion along direction of wave (longitudinal) e.g., sound
  - perpendicular to direction (transverse) e.g., waves on string, light
- Wave motion described by function of both position and time

Transverse and Longitudinal Waves

- *Transverse Wave*



- *Longitudinal Wave*



SG For waves traveling along the fabric band stretched across the classroom, if I pull on the band with a greater tension before plucking it, will the resulting wave speed be:

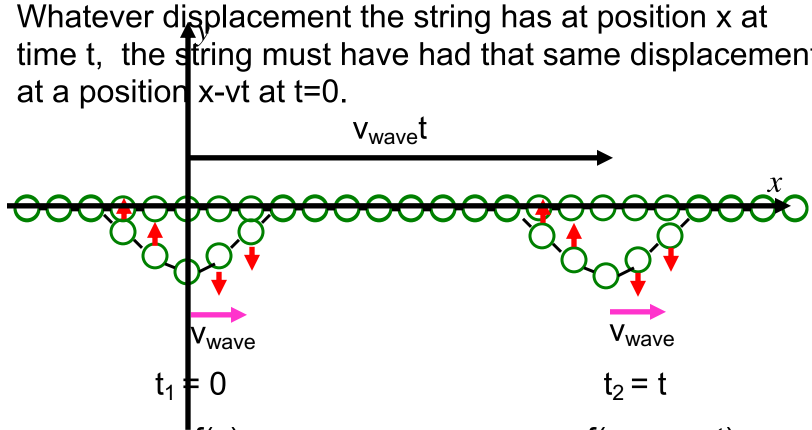
1. Greater?
2. Smaller?
3. Unchanged?
4. Can't tell

Speed of wave

- wave on string -- how quickly does disturbance propagate from one point to another?
  – wave speed, v
- expect it depends on tension and mass density of string μ (mass per unit length)
- dimensional analysis:

Mathematical Description of Traveling Wave

Whatever displacement the string has at position x at time t, the string must have had that same displacement at a position x-vt at t=0.



$y = f(x - vt)$ Wave traveling in $+x$ direction

$y = f(x + vt)$ Wave traveling in $-x$ direction

Mathematical description

- Describe wave by ***wave function*** which tells you the size of the wave at each point in space (x) and time (t)

$$y = f(\boldsymbol{x,t})$$

- Think of sinusoidal waves for simplicity

- At fixed point in space, have SHM:

$$y = \boldsymbol{A\cos(}\omega t) = A\cos(2\pi f t)$$

Description continued

- Not full story – the amplitude of wave depends on position as well as time

- Wave is collection of SHM oscillators where each oscillator has different ***phase $\theta(x)$***

$$y = \boldsymbol{A\cos(}\omega t - \theta') \quad \text{OR}$$

$$y = \boldsymbol{A\sin(}\omega t - \theta) \; \textit{\textbf{why?}}$$

Whatever displacement the string has at position x at time t, the string must have had that same displacement at a position x-vt at t=0.