

Solutions

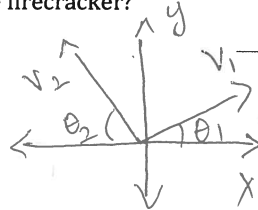
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Physics 215 Fall 2019
Practice midterm 3

1. [25 pts] A firecracker at rest on a concrete pad explodes into 3 fragments that **slide along the ground**. A "bird's eye" view of the velocities of two of the fragments are shown in the diagram below. Fragment 1 has a mass $m_1 = 2.0$ kg with speed $v_1 = 5.0$ m/s and angle $\theta_1 = 20^\circ$, and fragment 2 has a mass $m_2 = 3.0$ kg, speed $v_2 = 3.0$ m/s and angle $\theta_2 = 60^\circ$ with respect to the negative x-axis. Fragment 3 has a mass $m_3 = 2.0$ kg. You must show all of your work to receive full credit.

- a) [2 pts] What is the initial momentum of the firecracker?

$$P_i = 0$$



- b) [6 pts] Write an expression for the total momentum of the system after the explosion in terms of the fragment masses and velocities.

$$P_f^x = m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} \quad (\text{total momentum in } x\text{-direction})$$

$$P_f^y = m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y} \quad (\text{total in } y\text{-direction})$$

- c) [5 pts] What is the speed of the third fragment after the explosion?

$$0 = (2)(5 \cos 20^\circ) + (3)(-3.0 \cos 60^\circ) + v_{3x}$$

$$0 = (2)(5 \sin 20^\circ) + (3)(3 \sin 60^\circ) + v_{3y}$$

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2}$$

- d) [3 pts] Calculate the kinetic energy for m_3 immediately after the explosion.

$$K_3 = \frac{1}{2} m_3 v_3^2$$

- e) [5 pts] The coefficient of kinetic friction between each of the fragments and the ground is $\mu_k = 0.2$. Write an expression for the work done by friction as a function of the distance traveled for the third fragment.

$$W_{\text{friction}} = -F_f d_3 \quad d_3 \equiv \text{distance covered by third fragment}$$

- f) [4 pts] Use the work-kinetic energy theorem to calculate the total distance traveled by the third fragment.

$$W_{\text{friction}} = K_3^f - K_3^i$$

$$+ F_f d_3 = + \frac{1}{2} m_3 v_3^2$$

$$d_3 = \frac{v_3^2}{2\mu_k g}$$

$$F_f = \mu_k mg$$

$$= \mu_k mg$$

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3. [25 pts] A basketball player leaps onto the scorer's table to try and save a ball that is going out of bounds. The player has mass $m = 60$ kg, and the table is 0.8 m high. To execute the leap, the player starts from rest and pushes against the ground for 0.2 s (the "takeoff period"). Assume that the player just barely makes it to the top of the table without overshooting it. *You must show your work to receive credit.*

a) [5 pts] What is the speed of the player immediately after leaving the ground?

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 \\ v &= \sqrt{2gh} \\ v &= \sqrt{2(9.81)(0.8)} \end{aligned}$$

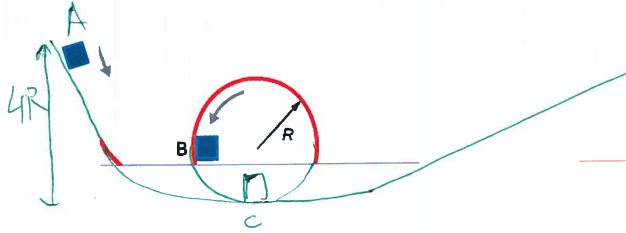
b) [5pts] What is the change in kinetic energy of the player during the takeoff period?

$$\begin{aligned} \Delta(K.E) &= \frac{1}{2}mv^2 - 0 \\ &= \frac{1}{2}(60)(2 \times 9.81 \times 0.8) \end{aligned}$$

c) [5 pts] What is the work done during the takeoff period?

$$\begin{aligned} \text{Work done} &= \text{Change in kinetic} \\ &\quad \text{energy} \\ &= \frac{1}{2}(60)(2 \times 9.81 \times 0.8) \end{aligned}$$

2. [25 pts] A small block of mass m slides without friction around the loop-the-loop apparatus shown to the right. Assume the block starts from rest at A. After the block passes through the loop, it begins to move up a ramp with an angle θ with respect to horizontal, as shown in the diagram above. The material on the ramp has a coefficient of friction μ_k . You should write all your answers only in terms of m , R , g , μ_k , and θ whenever applicable.



- a) [5 pts] Write an expression for the speed of the block at B.

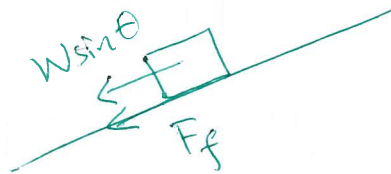
$$\frac{1}{2}mv_B^2 + mgh_B = mgh_A$$

$$\Rightarrow v_B = \sqrt{2g(h_A - h_B)} = \sqrt{6gR}$$

- b) [5 pts] Write an expression for the force of the track on the block at B.

$$F_c = \frac{mv_B^2}{R} = 6mg$$

- c) [8 pts] Draw a free body diagram for the block as it moves across the rough ramp surface, and use it to write an expression for the net work done on the block as it slides up the ramp as a function of the distance traveled along the ramp.



$$W_{\text{net}} = -(W \sin \theta + F_f)d$$

where d is the distance travelled along the ramp.

- d) [7 pts] How far will the block slide before coming to rest?

$$v_c = \sqrt{8gR} = \sqrt{2g(h_A - h_c)}$$

$$W_{\text{net}} = K_f - K_i$$

$$-(W \sin \theta + \mu_k W \cos \theta)d = -\frac{1}{2}mv_c^2$$

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d) [5 pts] What is the impulse during the takeoff period?

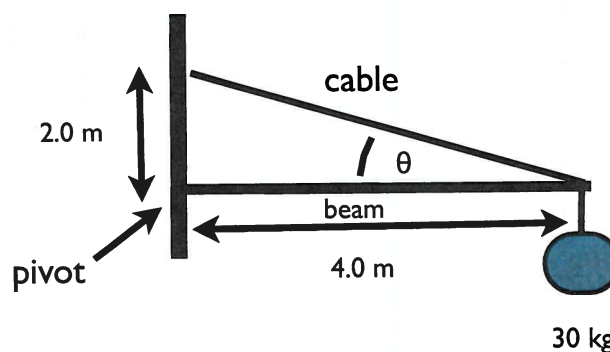
$$\begin{aligned} I &= F_{\text{avg}} \Delta t = m \Delta v \\ &= (60) (\sqrt{2(9.81)(0.8)}) \end{aligned}$$

e) [5 pts] What is the average power during the takeoff period?

$$P = \frac{\text{Work done}}{\Delta t} = \frac{\frac{1}{2} (60) (2 \times 9.81 \times 0.8)}{0.2}$$

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4. [25 pts] A uniform beam with mass 25 kg is attached at one end to a pivot point on a vertical surface and is held in a horizontal position by a cable as shown in the diagram. A weight of mass 30 kg is hung from the end of the beam.



- a) [3 pts] Use simple trigonometry to find the angle θ

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

- b) [9 pts] At what distance is the center of mass of the (beam+weight) system from the weight?

$$x_{\text{cm}} = \frac{(25)(2) + (30)(4)}{30 + 25}$$

- c) [5 pts] If the system is to remain in rotational equilibrium what condition must be satisfied?

$$\vec{\tau}_{\text{net}} = 0$$

$$\vec{F}_{\text{net}} = 0$$

- d) [8 pts] Calculate the tension in the cable if the system is at rest.

1. $(T \sin \theta)(4) = (30)(9.81)(4) + (25)(9.81)(2)$

$$T = (2)(30)(9.81) + (25)(9.81)$$