Name: Solutions

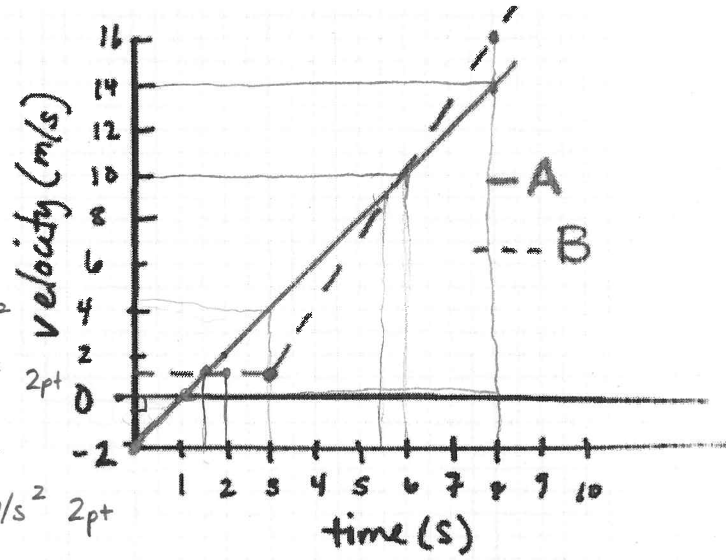
1. [25 pts] Two trains A and B move along straight and parallel tracks. Their motions are represented in the following velocity-vs. time graph. Use the graph to answer the following questions. *You must show all work to receive credit.*

a) [4 pts] At what (approximate) times(s) do the two objects have the same velocity?

2 pts $t = 1.5\,s$

2 pts $t = 6\,s$

OR 2 pts when velocities are same on this plot



b) [6 pts] Compute the average acceleration of both A and B in the time interval between t=2 and t=6 seconds.

A: $a_{av} = \frac{v_f - v_i}{t_f - t_i} = \frac{10\,m/s - 2\,m/s}{6s - 2s} = \frac{8}{4}\,m/s^2 = 2\,m/s^2$ 2pt

2pt

B: $a_{av} = \frac{v_f - v_i}{t_f - t_i} = \frac{10\,m/s - 1\,m/s}{6 - 2s} = \frac{9}{4}\,m/s^2 = 2.25\,m/s^2$ 2pt

c) [3 pts] Calculate the instantaneous acceleration of B at t=4 seconds.

$a_{inst} = \text{slope} = \frac{(16-1)\,m/s}{(8-3)\,s} = \frac{15}{5}\,m/s^2 = 3\,m/s^2$

1 pt 1 pt 1 pt

d) [6pts] What is the net displacement of A between t=0 and t=8 seconds? How much distance has it traveled during this interval?

5 pts

displacement = area under curve

$\underbrace{-\tfrac{1}{2}\cdot 2\,\tfrac{m}{s}\cdot 1\,s}_{-1\,m} + \underbrace{\tfrac{1}{2}\,14\,m/s\cdot 7\,s}_{49\,m} = 48\,m$

distance $|-1| + |49|\,m = 50\,m$

e) ~~[5 pts] What is the net displacement of A between t=0 and t=8 seconds?~~ B

or $v = -2\,m/s + 2\,m/s^2\,t \Rightarrow$

$x = -2\,m/s\,t + 1\,m/s^2\,t^2 + x_0$

$\Rightarrow x - x_0 = -2\,m/s\,t + t^2 (1\,m/s^2)$

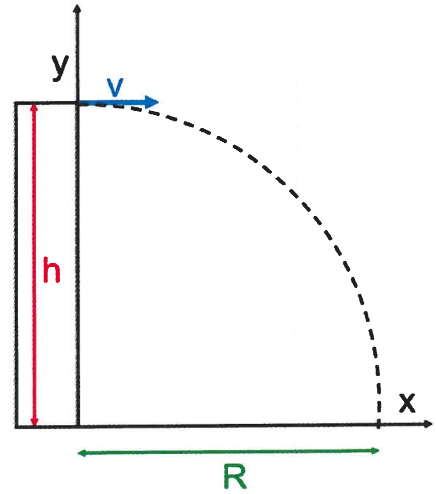
$\Delta s = |-2\,m/s\,(1s) + 1\,m/s^2 (1s)^2| +$

$\Rightarrow \Delta x = -2\,m/s(8s) + 1\,m/s^2 (8s)^2$

$\Delta x = -16 + 64 = 48\,m$

Name: ______________________________

2. [25 pts] A physics professor decides to throw a melon horizonally from the roof of the physics building at a speed 0.5m/s as shown in the picture below. The building is 12 m high and the melon lands a distance R from the base of the building. *You must show all work to receive credit.*



a) [4 pts] What are the initial (t=0) components of the velocity vx and vy of the melon with respect to the x and y axes shown?

$V_x = 0.5 m/s$

$V_y = 0.$

b) [4 pts] Write down equations showing how those velocity components change with time t:

$V_x = 0.5 m/s \qquad V_y = 0 - gt.$

$V_x = 0.5 m/s \quad V_y = -gt$

c) [4 pts] Write down equation showing hoe the x- and y- coordinate of the melon change with time t:

$x = 0.5t.$

$y = h - \frac{1}{2}gt^2.$

d) [6 pts] Calculated the time at which the melon hits the ground. Find also the distance R from the building where this occurs.

set $y = 0.$

$t = \sqrt{\frac{2h}{g}}$

$= \sqrt{\frac{(12)(2)}{10}} = \sqrt{2.4}.$

$= 1.54 \text{ sec}$

$R = V_x t.$

$= 0.5\sqrt{\frac{2h}{g}} = 0.5\sqrt{2.4}$

$= 0.77m$

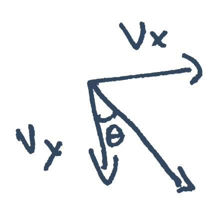
e) [7 pts] Calculated the speed of the melon the instant before it hits the ground, and also the angle its velocity vector makes with the downward vertical.

$V_x = 0.5 m/s.$

$V_y = -10(\sqrt{2.4}) = -10(1.54).$

$= -15.4.$

$\text{speed} = \sqrt{V_x^2 + V_y^2}$



$\tan\theta = \frac{V_x}{V_y}$

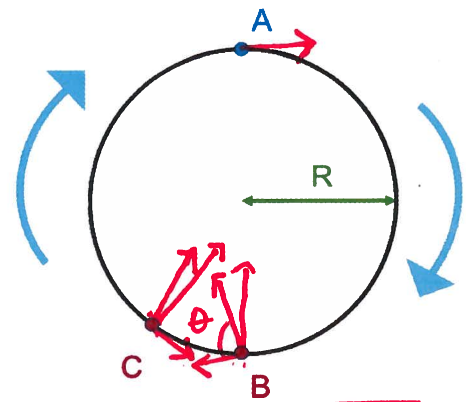
$\Rightarrow \theta = \tan^{-1}\left(\frac{V_x}{V_y}\right) = \tan^{-1}\left(\frac{0.5}{15.4}\right)$

$= 1.85°$

Name: ______________________________

3. [25 pts] A car starts from rest at point A on the diagram shown, and accelerates in a clockwise direction around a circular track of radius 100m in such a way as to increase its speed by 6m/s every 2 seconds. *You must show all work to receive credit.*

a) [3 pts] On the diagram, draw a vector representing the initial acceleration. What is its magnitude?



$a = \frac{6}{2} = 3 m/s^2.$

b) [4 pts] Using the constant acceleration equation for the tangential component of the motion, calculate the time for the car to complete half the circuit (e.g. go around half the circle) and reach point B.

$\Delta s = \frac{2\pi R}{2} = \pi R.$

$\Delta s = \frac{1}{2} a t^2. \Rightarrow t = \sqrt{\frac{2 \Delta s}{a}} = \sqrt{\frac{2\pi(100)}{3}}$

$= 14.47 sec.$

c) [4 pts] Calculated the speed of the car, and the magnitude of the radial acceleration at B.

$v_f = v_0 + at = 0 + (3)(14.47) = 43.41 m/s.$

$a_r = v^2/r = 18.84 m/s^2.$

d) [10 pts] What is the magnitude of the total acceleration at B. Draw an arrow to indicate its direction, label the angle in your diagram, and state the value of the angle in degrees.

$\tan\theta = \frac{a_r}{a_t}$

$\theta = \tan^{-1}\left(\frac{18.84}{3}\right)$

$\theta = 80^\circ.$

$a_{total} = \sqrt{a_t^2 + a_r^2}$

$= \sqrt{3^2 + (18.84)^2} = 19.08 m/s.$

e) [5 pts] Just beyond B, the car applies its brakes and starts to lose speed at a rate of 4 m/s every second. Compute its speed 5 seconds after the breaks were applied, at point C (not drawn precisely to scale).

$v_f = 43.41 - 4(5) = 23.41 m/s.$

f) [5pts] Compute also the radial and tangential components of the acceleration at C. Show the approximate direction of the total acceleration at C on the diagram.

$a_r = \frac{(23.41)^2}{100} = 5.48 m/s^2.$

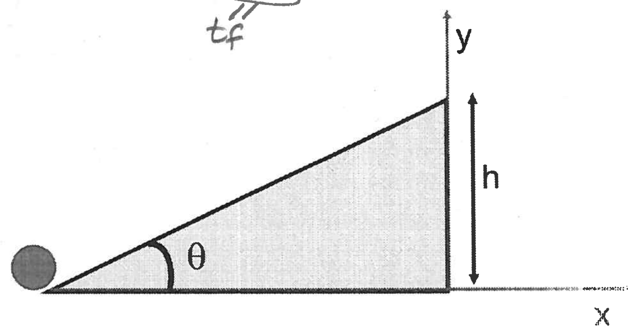
$a_t = -4 m/s^2.$

Name: solutions

$$t = \frac{-v_i \pm \sqrt{v_i^2 - 4(-\frac{1}{2}a)(-\ell)}}{2(-\frac{1}{2}a)} = \frac{+3 \pm \sqrt{9 - 4 \cdot 2.5 \cdot 0.5}}{+2(2.5)} = \frac{3 \pm 2}{5} = \boxed{0.2\,s}, \ 1\,s$$

t_f

4. [25 pts] A ball is projected with velocity 3 m/s up an inclined plane as shown in the diagram. The plan is inclines at an angle of θ =30 degrees with respect to the horizontal and rises vertically a total height of h= 0.25m. While moving up the plane, the ball suffers a deceleration of magnitude 5 m/s^2 due to gravitational effects. *You must show all work to receive credit.*



a) [6 pts] Calculate the length of the inclined plane. Use an equation to demonstrate that the ball reaches the top of the plane with a finite velocity.

2pt $\frac{h}{\ell} = \sin\theta \Rightarrow \ell = \frac{h}{\sin\theta} = 0.5\,m$

Along the plane

$\ell = 0 + v_i t - \frac{1}{2} a t^2$

$t = \frac{-v_i - \sqrt{v_i^2 - 4 \ldots}}{2(-\ell)}$

$v_f = v_i - at$

$v_f = (3 - 5 \cdot \frac{1}{5})$

$\boxed{v_f = 2\,m/s}$

4pts

b) [4pts] At the top of the plane, the ball becomes a projectile. Calculate the x- and y- components of the ball's velocity at the point where it leave the plane.

2 m/s, θ

2pt $v_{yi} = v_i \sin\theta = 2\,m/s \sin 30° = 1\,m/s$

2pt $v_{xi} = v_i \cos\theta = 2\,m/s \cos 30° = 1.73\,m/s$

c) [5 pts] How much time will have elapsed after leaving the plane before the ball attains its maximum height?

3pts $v_y = v_{yi} + a_y t$

$0 = 1\,m/s - 9.8\,m/s^2\, t \Rightarrow \boxed{t_m = 0.1\,s}$ 2pt

d) [5 pts] Calculate the time taken for the ball to return to the ground from this highest point.

2pt ground: $y = 0$

$0 = h + v_{iy} t_g - \frac{1}{2} a_y t_g^2 \Rightarrow t_g = \frac{-v_{iy} \pm \sqrt{v_{iy}^2 - 4 \cdot h \cdot \frac{1}{2} a_y}}{-2(+\frac{1}{2} a_y)}$ 2pts

$t_g = \frac{+1\,m/s \pm \sqrt{1 + (4)(\frac{1}{4})\,4.9}}{+9.8} = 0.35\,s$

1 pt $\Delta t = t_g - t_m = 0.25\,s$

e) [5 pts] How far horizontally R will the ball have travelled from the end of the plane to the point where it hits the ground?

$\underbrace{R = v_{ix}\, t_g}_{4\,pts} = 1.73\,m/s \cdot 0.35\,s = \boxed{0.61\,m}$ 1pt