Name: Solutions

1. [25pts] A spaceship is moving through vacuum at a significant fraction of the speed of light.

a. [5pts] A clock in the spaceship moves along the x-axis in your reference frame at a speed of 0.80c and reads zero as it passes the origin. As it passes the 180 meter mark on your axis, how much time has passed in your reference frame?

225/c

$$\Delta t' = \frac{180}{0.8 \times 3 \times 10^8} = 7.5 \times 10^{-7} \text{ sec.}$$

This is the time in your frame

b. [5pts] What time does the clock on the spaceship read when it passes the 180 meter mark (on your axis)?

$$\Delta t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Delta t$$

where Δt is the time in clock's frame

$$\Rightarrow \Delta t = (0.6)(7.5 \times 10^{-7})$$

4.5×10^{-7}

c. [5pts] The spaceship changes its speed. It is now moving at such a speed in your frame that its measured length is one-third its proper length. How fast is the spaceship moving relative to your frame?

$$L = \sqrt{1-\frac{v^2}{c^2}}\, L_0$$

$$L = \frac{1}{3} L_0$$

$$\Rightarrow v = 0.94c.$$

$\frac{2\sqrt{2}}{3}c$ or 2.8×10^8 m/s

L_0 is the proper length of space ship in it's rest frame

d. [5pts] At the end of the trip, the spaceship lands on a flat landing strip, hits the brakes and slows from a velocity of 100 m/s to 5 m/s due to a constant frictional force between the ship and the ground. Assume the tires are skidding, so that there is no rolling friction and the friction is kinetic. If the ground is flat with a coefficient of kinetic friction between the ground and the tires of 0.2, and the mass of the spaceship is M=1000 kg, write an expression for the. distance ship travels in slowing down from 100 m/s to 5 m/s

$$\Delta s = \frac{5^2 - 100^2}{2a} = 2542.04 \text{ m.}$$

where $a = \frac{(0.2)(1000)(9.81)}{1000}$

$a = 1.962$

e. [5pts] After it slows to 5 m/s, the ship bumps into a tug cart of mass 500 kg, and the ship and the cart stick together. Neglecting friction, what is the speed of the ship+ cart system after they bump into each other?

$$P_i = (5)(1000).$$

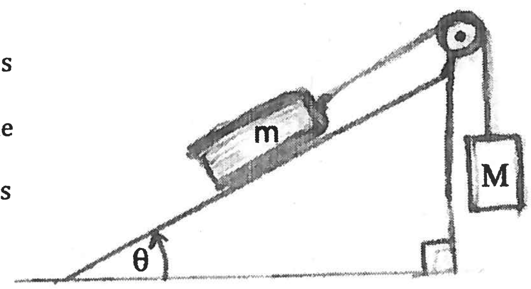
$$P_f = (1500) v_f.$$

$$P_i = P_f.$$

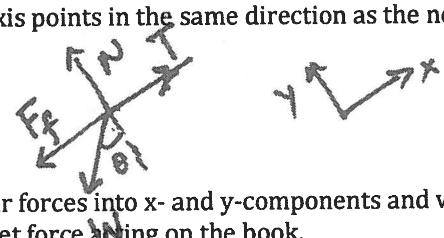
$$v_f = \frac{(5)(1000)}{1500}$$

$$v_f = 3.33 \text{ m/s.}$$

2. [25 pts] The figure to the right shows a book of mass m resting on a slope with $\theta = 15$ degrees. The book has coefficients of friction $\mu_s = 0.3$ and $\mu_k = 0.1$ with the surface. It is connected via a massless string over a massless, frictionless pulley to a hanging block of mass M= 2.0 kg.



a) [6 pts] Assume the book does not slip. Draw a free body diagram for the book. Choose your axes so that the y-axis points in the same direction as the normal force on the book.



b) [6 pts] Resolve your forces into x- and y-components and write down Newton's 2nd law for each component of the net force acting on the book.

For x: $T - F_f^s - W\sin\theta = ma_x$

For x: $N - W\cos\theta = ma_y$.

where F_f^s = static friction

c) [4 pts] Draw a free body diagram for the hanging block. Choose the positive y-axis to point upwards. Write down Newton's 2nd law for the block.

$T - W = ma_y$

d) [2 pts] Write an expression for the frictional force acting on the book in terms of the normal force on the book.

Since the book doesn't slip we have purely static friction $F_f^s \le \mu_s N$.

e) [7 pts] Using the equations derived in parts a), b) and c), calculate the minimum mass m that the book can have so that the book will stick and not slip. Show your work.

$a_x = 0 = a_y$. for m

$a_y = 0$ for M $\Rightarrow$ $T = Mg$.

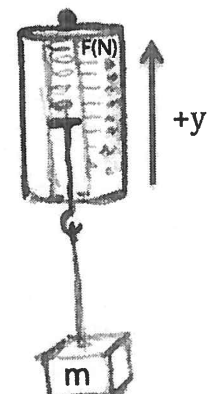
$\Rightarrow F_f^s = T - W\sin\theta$.

$\Rightarrow T - W\sin\theta \le \mu_s N$.

$Mg - mg\sin\theta \le \mu_s mg\cos\theta$.

$\Rightarrow m \ge \frac{M}{\mu_s\cos\theta + \sin\theta}$

3. [25 pts] A demo has been installed in an elevator in a very tall physics building. In the demo, a mass m= 0.7 kg hangs by a string from a force probe that measures the tension in the string, as shown in the figure. The force probe measures a positive force when the string exerts an upward tension on the mass. *You must show all of your work.*



a) [3 pts] Initially, the elevator is at rest. Draw a free body diagram for the mass m. Include axes.

↑T ↓W where T is the force measured by force probe. ↑y-axis.

b) [4pts] **Use Newton's second law** to calculate the reading on the force probe (in N) while the elevator is at rest.

$T - W = ma_y \quad a_y = 0 \quad T = mg = (0.7)(9.81$

c) [5 pts] Now the elevator accelerates upward at a constant rate of $a_1 = 2.1\ m/s^2$. Draw a free body diagram for mass m. **Use Newton's second law** to calculate the reading on the force probe (in N) in this new situation.

$T - W = ma_y \Rightarrow T = ma_y + mg$

$T = (0.7)(2.1) + (0.7)(9.8$

d) [4 pts] After a brief period of acceleration, the elevator begins moving upward at a constant speed of v = 7 m/s . **Use Newton's second law** to calculate the reading on the force probe(in N) while the elevator moves at constant speed.

$\Delta v = 0 \Rightarrow a_y = 0.$

$T = W = (0.7)(9.81)$ N.

e) [4 pts] While still moving upward, the elevator decelerates at a rate $a_2 = -3.3\ m/s^2$. **Use Newton's second law** to calculate the reading on the force probe (in N) during this deceleration.

$T = ma_y + mg \mid a_y = -3.3\ m/s^2$

$T = (0.7)(-3.3) + (0.7)(9$

f) [5 pts] After the elevator stops, a professor accidentally bumps into the force probe and knocks it off the wall, so that both the mass and the force probe fall to the ground together in free fall. While they are both falling, draw a free body diagram for the mass. What is the reading on the force probe (in N)? Neglect air resistance.

$T = ma_y + mg \qquad a_y = -g.$ for free-fall

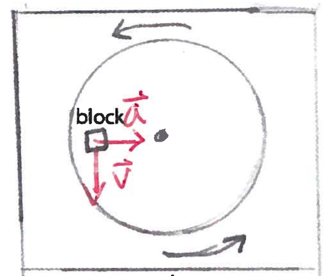
$= -mg + mg = 0.$

$\Rightarrow$ force probe reads zero.

Name: ______________________________

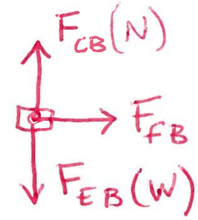
4. [25 pts] At the Destiny USA mall, a child accidently leaves his toy block on the floor of the carousel (a ride where children sit on wooden animals attached to a rotating disk). The carousel rotates counterclockwise at a constant rate. The block does not slip on the floor of the carousel. *You must show your work to receive credit.*

a) [4 pts] In the **top view** diagram at right, draw and label arrows representing the linear velocity and acceleration vectors for the toy block.



top view

b) [6 pts] Draw a free body diagram for the toy block pictured from a **side view**. Label your forces with 2 subscripts indicating which body the force acts on and which body is supplying the force.



c) [6 pts] The mass of the block is m= 0.4 kg and its linear speed relative to the earth is v = 3.0 m/s. The radius of the circle through which the block moves is r_1 = 3.1 m and the coefficient of static friction between the block and the carousel is μ_s= 0.5. Find the magnitude of the net force on the block. Show your work.

$$F_{net} = \frac{mv^2}{r_1} = \frac{(0.4\ kg)(3.0\ m/s)^2}{3.1\ m} = 1.16\ N$$

note, this is smaller than $F_{s,max}$ at this distance

d) [3 pts] Now the block is placed so that the radius of the path is r_2= 6.1 m. The carousel is still rotating with the same constant angular velocity. What is the new linear velocity of the block?

$V = \omega r$

$\omega = \frac{v_1}{r_1} = 0.96\ rad/s$

$V_2 = \omega r_2 = 5.9\ m/s$

e) [6 pts] For the conditions in part d), will the block slip? Show your work.

$$F_{fs} \le \mu_s N = \mu_s mg = F_{f,max} = 1.96N$$

$$F_{Net,circle} = \frac{mv_2^2}{r_2} = 2.28N$$

$F_{Net} > F_{f,max}$ so block slips.