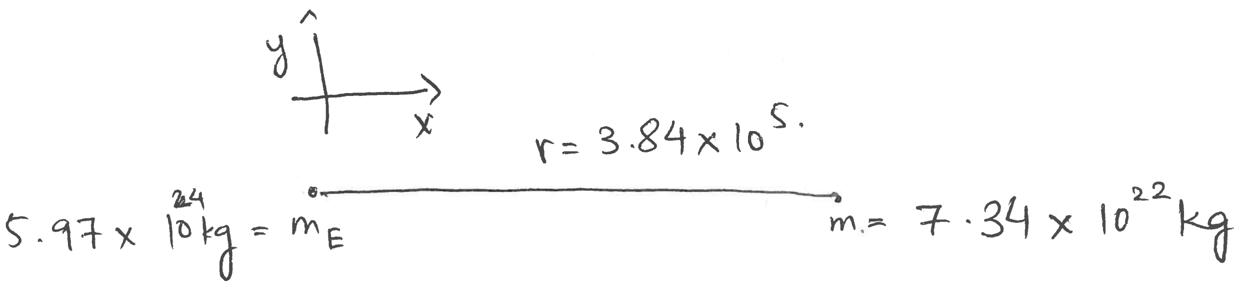
Homework (Week-11) Solutions:

Ch-9: Problem 111

1% of Earth's mass $= 0.01 \times 5.97 \times 10^{24}$ kg

$= 5.97 \times 10^{22}$ kg.



$$X_{cm} = \frac{m_E(0) + m(3.84 \times 10^5)}{m_E + m}$$

$$= \frac{(7.34 \times 10^{22})(3.84 \times 10^5)}{5.97 \times 10^{24} + 7.34 \times 10^{22}}$$

if 1% mass moved to moon; new mass of moon

$m = (5.97 + 7.34) \times 10^{22}$ kg.

$m_E = 5.9103 \times 10^{24}$ kg.

$$X'_{cm} = \frac{(13.31 \times 10^{22})(3.84 \times 10^5)}{5.9103 \times 10^{24} + 13.31 \times 10^{22}}$$

Other method without any number.

$$X_{cm} = \frac{mr}{m_E + m}.$$

$$X'_{cm} = \frac{\left(m + (0.01)\, m_E\right) r}{(m_E - 0.01 m_E) + (m + 0.01 m_E)}.$$

$$= \frac{mr}{m_E + m} + \frac{0.01\, m_E\, r}{m_E + m}.$$

$$X'_{cm} = X_{cm} + (0.01r)(0.98).$$

$$\frac{m_E}{m_E + m} \sim 0.98 \simeq 1.$$

$$= X_{cm} + 0.0098 r.$$

$$X'_{cm} = X_{cm} + 3793.36$$

centre of mass will move by almost ~ 3.8 km.

Ch- 10

Problem 47:

$$\omega = 2.5 \text{ rad/s}$$

$$F_s \leq \mu_s N$$

$$F_s = \frac{mv^2}{r} = F_c$$
$$= m\omega^2 r$$

$$m\omega^2 r \leq \mu_s N$$
$$r \leq \frac{\mu_s N}{m\omega^2}$$
$$r \leq \frac{\mu_s \cancel{m} g}{\cancel{m}\omega^2}$$
$$r \leq \frac{\mu_s g}{\omega^2}$$
$$r \leq \frac{(0.5)(10)}{(2.5)^2}$$

$$r \leq 0.8 m$$

Problem 56:

$$K.E_{rot} = \frac{1}{2} I\omega^2 \qquad I = \frac{1}{2} m(r_1^2 + r_2^2)$$

r_1 = inner radius
r_2 = outer radius

Problem 68:

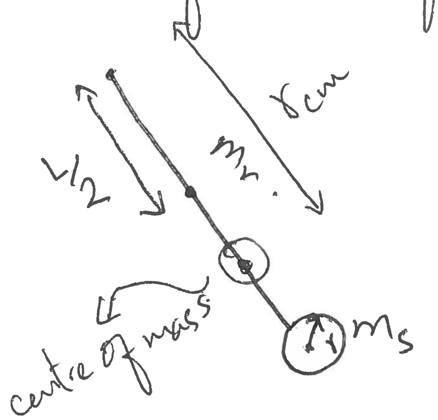
Moment of inertia of the rod+ sphere system

$$I = \frac{1}{3} m_r L^2 + \frac{2}{5} m_s r^2 + m_s (L+r)^2.$$

$m_r \equiv$ mass of rod.

$m_s \equiv$ mass of sphere.

Centre of mass of the system. along the rod



$$r_{cm} = \frac{m_r\left(\frac{L}{2}\right) + m_s (L+r)}{m_r + m_s}.$$

At rest the system has purely gravitational potential energy

At lowest point, CM of the system drops a height h of

$$h = r_{cm}(1-\cos\theta).$$

Using conservation of energy.

$$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = mgh.$$

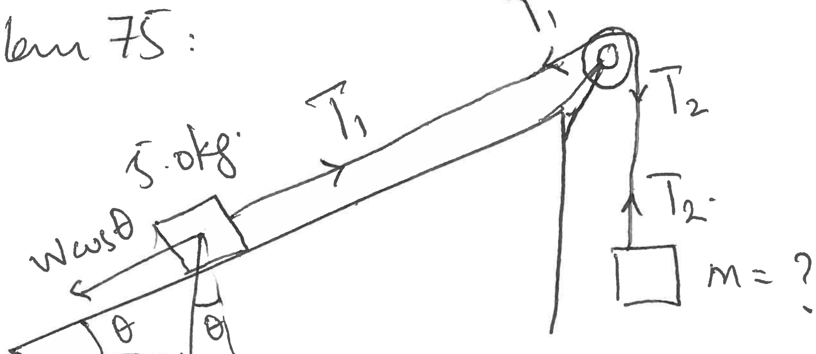
$$v_{cm} = \omega r_{cm}.$$

$$\frac{1}{2}m\omega^2 r_{cm}^2 + \frac{1}{2}I\omega^2 = mgr_{cm}(1-\cos\theta).$$

where m is mass of rod + sphere.

rest is plugging in values!

Problem 75:



$T_1 = m_1 g \cos\theta$

for 5.0kg mass. m_1 $\qquad$ $T_2 = m_2 g$

$T_1 - m_1 g \cos\theta = m_1 a$ $\qquad$ $T_1 r_1 = T_2 r_2$

$r_1 m_1 g \cos\theta = m_2 g \, r_2$

$m_2 g - T_2 = m_2 a$ $\qquad$ $m_2 = ?$

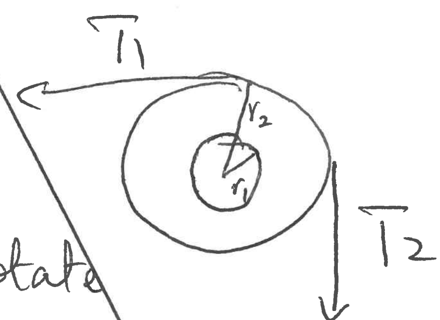
$m_2 = \dfrac{r_1 m_1 \cos\theta}{r_2}$

$(T_2 - T_1)(r_2 - r_1) = I\alpha$ $\qquad$ $\alpha = \dfrac{a}{r_2 - r_1}$

$m_2 = \dfrac{0.2}{0.3} \, 5 \dfrac{\sqrt{3}}{2}$

where r_2 : outer radius

r_1 : inner radius



for the pulley to not rotate

$\alpha = 0 \Rightarrow a = 0$

$T_1 - T_2 = m_1 g \cos\theta - m_2 g + (m_1 + m a)$

$\left(\dfrac{T_1 - T_2 + m_2 g - m_1 g \cos\theta}{m_1 + m_2} \right) = a$

$$(T_2 - T_1)(r_2 - r_1) = \frac{I a}{(r_2 - r_1)}$$

$$(T_2 - T_1) = \frac{I}{(r_2 - r_1)^2} \left[T_1 - T_2 + m_2 g \right.$$

$$T_1 - m_1 g \cos\theta' = m_1 a \qquad \theta' = 90 - \theta$$

$$m_2 g - T_2 = m_2 a.$$

$$(T_2 r_2 - T_1 r_1) = I \alpha.$$

r_1 : inner radius
r_2 : outer radius.

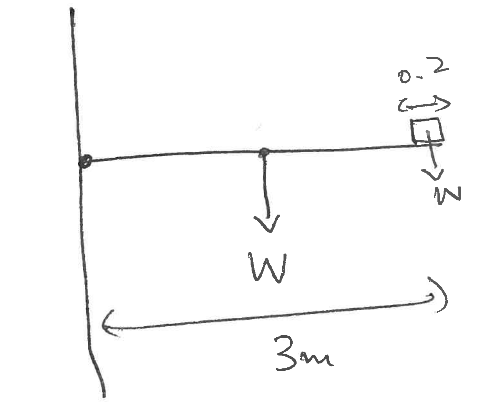
$$\alpha = 0 \Rightarrow a = 0$$

$$-(m_1 g \cos\theta') r_1 + m_2 g r_2 = 0$$

$$\Rightarrow m_2 = m_1 \sin\theta \cdot \frac{r_1}{r_2}$$

$$m_2 = 1.666.67 \, kg.$$

Problem 81:



$$\text{Clockwise torque} = W(1.5) + (1)(3 - 0.1)$$

$$= (2)(10)(1.5) + (1)(2.9)$$

$$= 30 + 2.9 = 32.9\ \text{Nm}$$

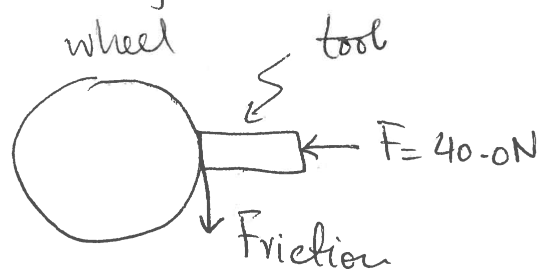
Problem 90:

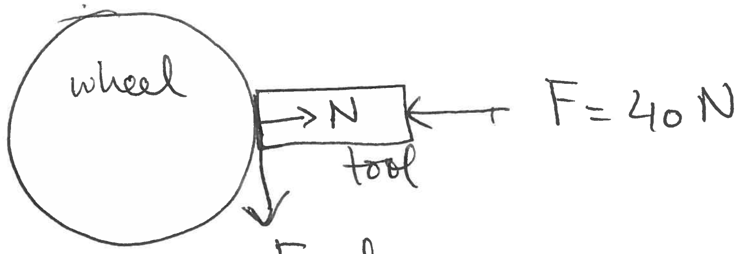
(a) $$\alpha = \frac{0^2 - \left(\frac{120 \times 2\pi}{60}\right)^2}{(2)(20)(2\pi)}$$

$$\alpha = \frac{-(120 \times 2\pi)^2}{(2)(20)(2\pi)(60)^2}$$

$$\tau = I_{\text{cylinder}}\,\alpha \qquad I_{\text{cylinder}} = \frac{1}{2}mr^2 \left(r = \frac{d}{2}\right)$$

(b)





Torque due to friction $= (40)(0.6)(1/2) = 12N$

if wheel has to rotate at constant angular velocity $\alpha = 0$

$\Rightarrow$ torque supplied by motor $= -12$ Nm

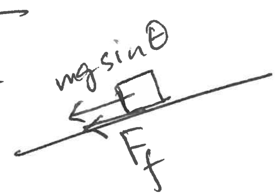
Counter clock-wise

Additional Problem.

(a) $U = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}(50)(0.03)^2$

(b) $\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$

$v = \sqrt{k(\Delta x)^2/m}$



(c) $F_f = \mu_k N$

$F_f = \mu_k mg\cos\theta$

(d) Work done by friction $= -(F_f)(d)$

(e) $E_i = \frac{1}{2} k (\Delta x)^2$

$E_f = mgh$ + Work done against friction.

$$h = \frac{1}{2} \frac{k(\Delta x)^2}{mg}$$

$$= \cancel{\left[\frac{\frac{1}{2}(50)(0.03)^2}{\frac{10}{1000} \cdot 10} + (0.3)\left(\frac{10}{1000}\right)(10)\cos(30^\circ) \times \frac{h}{\sin\theta} \right]}$$

$\Rightarrow$

$mgh = \frac{1}{2} k (\Delta x)^2 -$ Work done against friction.

Use this to find h.

If the block gains a height h, it moves a distance $h/\sin\theta$ along the ramp.

$E_f = mgh$

$$mgh = \frac{1}{2} k (\Delta x)^2 - \mu_k mg \cos\theta \cdot \frac{h}{\sin\theta}.$$

use this to find if $h \geq 0.1$ m.

(f) Force on the cart $= \left(\frac{(210)}{1000}\right)\left(\frac{3}{2}\right)$

$$= \left(\frac{210}{1000}\right)\left(\frac{3}{2}\right)$$

distance covered by cart is:

$$\Delta s = \frac{1}{\cancel{2}}\left(\frac{3}{\cancel{2}}\right)(\cancel{2})^{\cancel{2}}$$

$$= 3 \text{ m}.$$

$$\text{Power} = \frac{\text{Work done on cart}}{\text{time taken}} = \frac{\frac{210}{1000} \cdot \frac{3}{2} \cdot 3}{2}$$

$$= \frac{210 \cdot 9}{4000}$$

(g) $$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$= \frac{(200)(3,2) + 10(5,4)}{210}$$

$$= \frac{(650, 440)}{210} = (3.09, 2.09)$$

$$(x_{cm}, y_{cm})$$