Welcome back to Physics 215

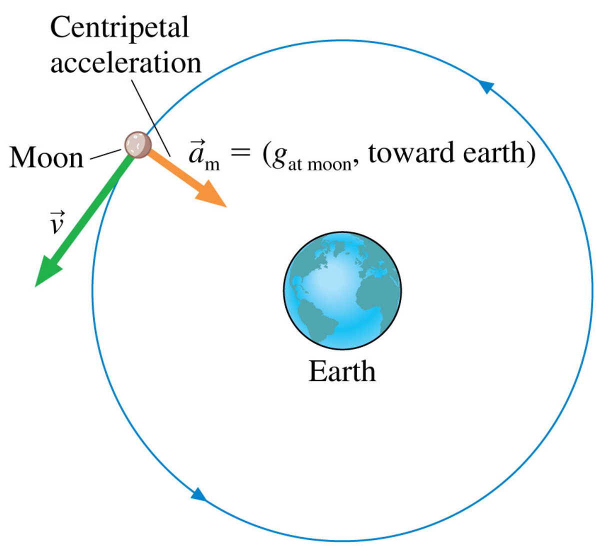
Today's agenda:

- *Kinetic energy, revisited*
- *Gravity!!*

Exam 3

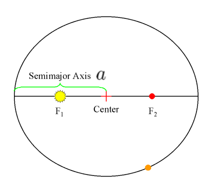
- Thursday, Nov 21st.
- In class
- **Covers Lecture 8-1 through 12-1:**
  - **Potential Energy through rolling without slipping**
- Must bring a calculator. There will be a formula sheet + table of moments of inertia.

The moon is in free fall around the earth.



Kepler's Laws experimental observations

3. Square of orbital period is proportional to cube of semimajor axis. $T^2 \sim a^3$



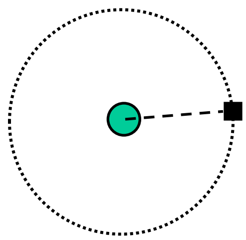
- We can deduce this (for ***circular orbit***) from gravitational law
- assume gravity responsible for acceleration in orbit →

Orbits of Satellites

- Following similar reasoning to Kepler's 3rd law →

$$GM_E M_{sat}/r^2 = M_{sat}v^2/r$$

$$v = (GM_E/r)^{1/2}$$



Gravitational Field

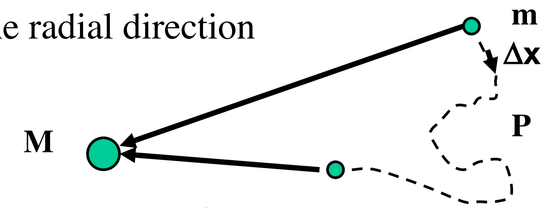
- Newton never believed in ***action at a distance***
- Physicists circumvented this problem by using new approach – imagine that every mass creates a ***gravitational field*** Γ at every point in space around it
- Field tells the magnitude (and direction) of the gravitational force on some test mass placed at that position →$F = m_{test}\Gamma$
- Close to earth: Γ=

Gravitational Potential Energy

Work done moving small mass along path P

$W = \sum F \cdot \Delta x$

But F acts along the radial direction



Therefore, only component of F to do work is along r

$W = - \sum F(r) \Delta r$

Independent of P!

Gravitational Potential Energy

Define the gravitational potential energy U(r) of some mass m in the field of another M as the work done moving the mass m in from infinity to r

➔ $U = \sum F(r) \Delta r = -GMm/r$

SG A football is dropped from a height of 2 m. Does the football's gravitational potential energy increase or decrease ?

1. decreases
2. increases
3. stays the same
4. depends on the mass of football

Gravitational Potential Energy near Earth's surface

$U_h =$

For small h/R_E :
$\Delta U = (GM_E m/R_E^2)h = mgh$!! as we expect

Called the "flat earth approximation"

Recall: Energy conservation

- Consider mass moving in gravitational field of much larger mass M
- Energy conservation $\Delta E = 0$,

  where $E = K+U = 1/2mv^2 - GmM/r$
- Hence work done by gravitational field changes kinetic energy: $W = -\Delta U = \Delta K$
- Notice E < 0 if object ***bound***

Escape speed

- Object can just escape to infinite r if E=0

→ $(1/2)mv_{esc}^2 = GM_Em/R_E$

→ $v_{esc}^2 = 2GM_E/R_E$

- Magnitude ? 1.1×10^4 m/s on Earth
- What about on the moon ? Sun ?

Consequences for planets

- Planets with large escape velocities can retain light gas molecules, e.g. Earth has an atmosphere of oxygen, nitrogen
- Moon does not
- Conversely Jupiter, Sun manage to retain hydrogen

Black Holes

- Suppose light travels at speed c
- Turn argument about – is there a value of M/R for some star which will not allow light photons to escape ?
- Need $M/R = c^2/2G$ ➔ density = 10^{27} kg/m^3 for object with R = 1m approx
- Need very high densities – possible for collapsed stars

Review: How to solve gravity problems

- For circular motion under gravity (i.e. orbits) use:
  - $F_{cent} = F_g$
  - $mv^2/r = GMm/r^2$
  - v here is velocity along the circle (tangent to orbit)

- For escape velocity (or shooting stuff up into space) use:
  - $K_1 + U_1 = K_2 + U_2$, with $U = -GMm/r$ and $K = ½\, mv^2$
  - V here is the total velocity (often perpendicular to surface.)

Gravity Sample problem: The space shuttle, with a mass of 5 million kg, in a 300-km-high orbit wants to capture a smaller satellite (mass of 5,000 kg) for repairs. What is the speed of the shuttle? What is the speed of the satellite?

Sample problem: A less-than-successful inventor wants to launch small satellites into orbit by launching them straight up from the surface of the earth at a very high speed.

A) with what sped should he launch the satellite if it is to have a speed of 500 m/s at a height of 400 km? Ignore air resistance.
B) By what percentage would you answer be in error if you used a flat earth approximation (i.e. U = mgh)?

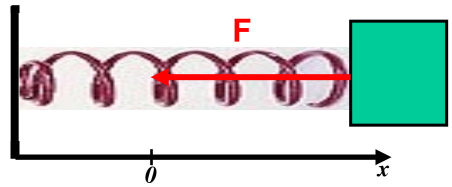
Oscillations

- *Restoring force* leads to oscillations about *stable equilibrium point*
- Consider a mass on a spring, or a pendulum
- Oscillatory phenomena also in many other physical systems...

Simple Harmonic Oscillator

$F_x = -k\,x$

Spring constant



Newton's 2nd Law for the block: $m\,a_x = F_x$

Differential equation for x(t) $$m\frac{d}{dt}\left(\frac{dx}{dt}\right) = -\,k\,x$$

Simple Harmonic Oscillator

Differential equation for x(t): $$m\frac{d}{dt}\left(\frac{dx}{dt}\right) = -\,k\,x$$

Solution: $x(t) = A\cos(\omega t + \phi)$

$$\frac{dx}{dt} =$$

$$\frac{d}{dt}\left(\frac{dx}{dt}\right) =$$

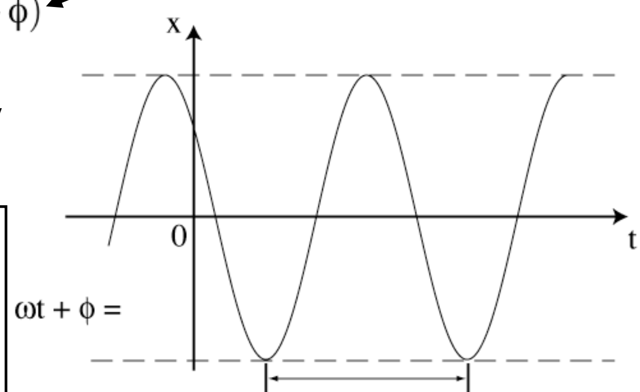
Demo: relationship between circular and simple harmonic motion

- What does ω mean?
- Circle: ω is the angular velocity
  - Period $T = 2\pi/\omega$
  - Projection is a sinusoidal motion!
- Simple harmonic motion: ω is the angular frequency
  - Also sinusoidal motion with SAME $T = 2\pi/\omega$

Simple Harmonic Oscillator

$$x(t) = A\cos(\omega t + \phi)$$

amplitude · **angular frequency** · **initial phase**



Units:
A - *m*
T - *s*
f - 1/s = Hz
(Hertz)
ω - rad/s

f – frequency
Number of oscillations per unit time

$$f = \frac{1}{T}$$

$$T = \frac{2\pi}{\omega}$$

T – *Period*
Time taken by one full oscillation

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Simple Harmonic Oscillator DEMO

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

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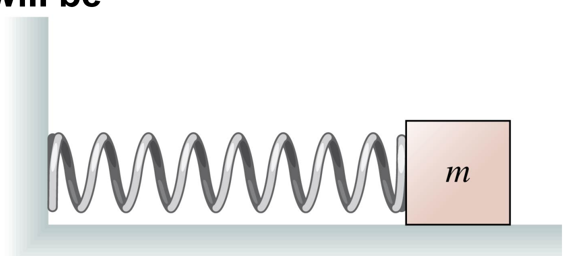
SG A mass oscillates on a horizontal spring with period $T = 2.0$ s. If the amplitude of the oscillation is doubled, the new period will be

A. 1.0 s.

B. 1.4 s.

C. 2.0 s.

D. 2.8 s.



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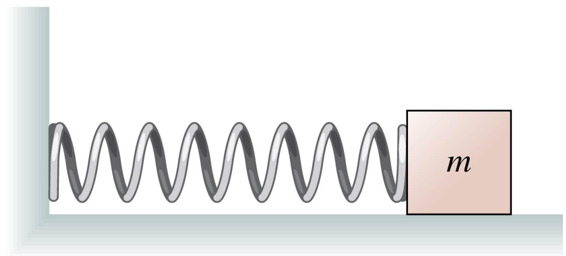
SG A block of mass *m* oscillates on a horizontal spring with period $T = 2.0$ s. If a second identical block is glued to the top of the first block, the new period will be

A. 1.0 s.

B. 1.4 s.

C. 2.0 s.

D. 2.8 s.



Simple Harmonic Oscillator Total Energy

$$E = U + K$$

$$E = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 \qquad v = \frac{dx}{dt}$$

$$E = \frac{1}{2}k[A\cos(\omega t + \phi)]^2 + \frac{1}{2}m[-A\omega\sin(\omega t + \phi)]^2$$

E =

Simple Harmonic Oscillator -- Summary

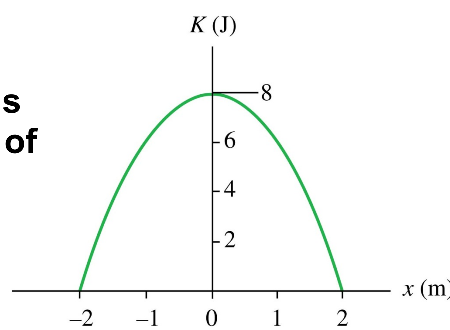
If **F = – k** ***x*** ***then***

$$x(t) = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{T}$$

$$E = \frac{1}{2}\,k\,A^2$$

SG A block oscillates on a very long horizontal spring. The graph shows the block's kinetic energy as a function of position. What is the spring constant?



A. 1 N/m.
B. 2 N/m.
C. 4 N/m.
D. 8 N/m.