

Welcome back to Physics 215

Today's agenda:

- *Kinetic energy, revisited*
- *Gravity!!*



Exam 3

- Thursday, Nov 21st.
- In class
- **Covers Lecture 8-1 through 12-1:**
 - **Potential Energy through rolling without slipping**
- Must bring a calculator. There will be a formula sheet + table of moments of inertia.

Kinetic energy of rolling

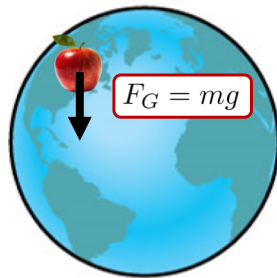
- The total kinetic energy of a rolling object is the sum of its rotational and translational kinetic energies:

SG. Two cylinders with the same radius and same total mass roll down a ramp. In cylinder A, a set of 8 point masses are equally spaced in a circle with radius r_1 around the cylinder's axis of rotation, while in cylinder B, the 8 point masses are a distance $r_2 > r_1$ from the center. Which cylinder reaches the bottom of the ramp first?

- A. Cylinder A
- B. Cylinder B
- C. They both reach the bottom at the same time
- D. Not enough information to tell

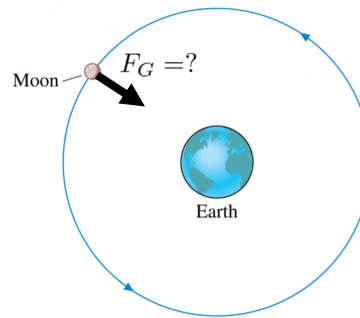
Gravity on earth

Anywhere on the surface of the earth (e.g. in Syracuse):



Earth

Still true elsewhere?



Gravity

- Before 1687, large amount of data collected on motion of planets and Moon (Copernicus, Galileo, Brahe, Kepler)
- Newton showed that this could all be understood with a new ***Law of Universal Gravitation***

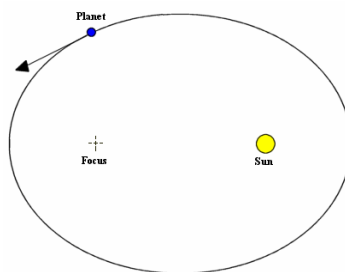


Gravity

- Before 1687, large amount of data collected on motion of planets and Moon (Copernicus, Galileo, Brahe, Kepler)
- Newton showed that this could all be understood with a new ***Law of Universal Gravitation***

Kepler's Laws experimental observations

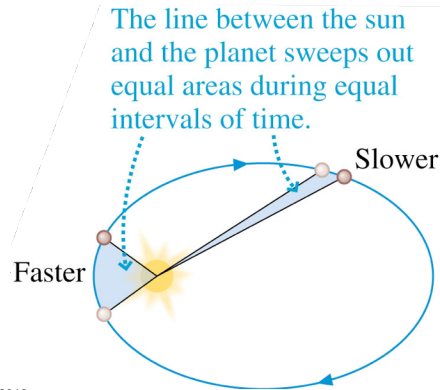
1. *Planets move on ellipses with the sun at one focus of the ellipse (actually, CM of sun + planet at focus).*



Kepler's Laws

experimental observations

2. A line from the sun to a given planet sweeps out equal areas in equal times.



*Conservation of angular momentum

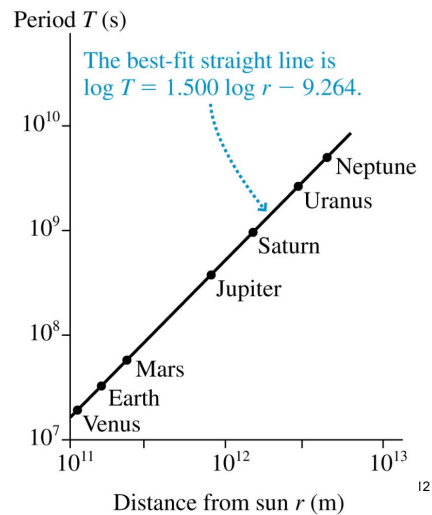
Physics 215 – Fall 2019

Lecture 12-2 11

SG The following is a log-log plot of the orbital period (T) compared to the distance to the sun (r). What is the relationship between T and r ?

1. $\log T = C r$
2. $T^2 = C r^3$
3. $T = C r$
4. $T^3 = C r^2$

For some constant C



Physics 215 – Fall 2019

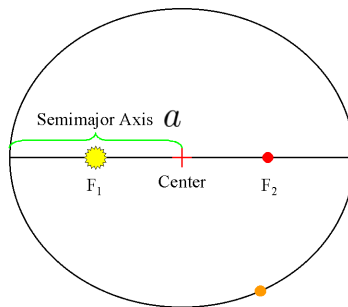
12

Kepler's Laws

experimental observations

3. *Square of orbital period is proportional to cube of semimajor axis.*

$$T^2 \sim a^3$$



Physics 215 – Fall 2019

Lecture 12-2 13

Universal Gravity

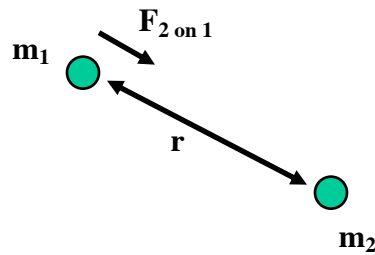
- Mathematical Principles of Natural Philosophy:

Every particle in the Universe attracts every other with a force that is directly proportional to their masses and inversely proportional to the square of the distance between them.

Physics 215 – Fall 2019

Lecture 12-2 14

Inverse square law



Physics 215 – Fall 2019

Lecture 12-2 15

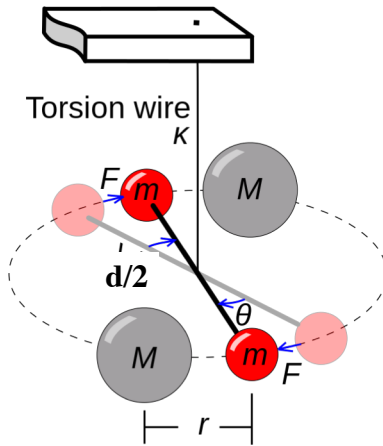
Interpretation

- F acts along line between bodies
- $F_{12} = -F_{21}$ in accord with Newton's Third Law
- **Acts at a distance** (even through a vacuum) ...
- G is a universal constant = $6.7 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$

Physics 215 – Fall 2019

Lecture 12-2 16

How to measure G?

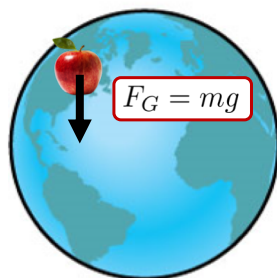


Physics 215 – Fall 2019

Lecture 12-2 17

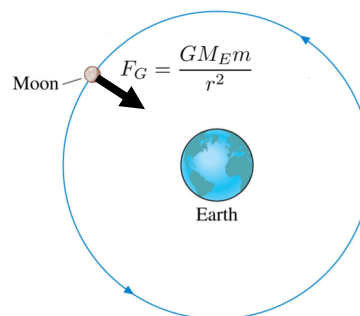
Gravity on earth

Anywhere on the surface of the earth
(e.g. in Syracuse):



Earth

Still true elsewhere?



Physics 215 – Fall 2019

Lecture 12-2 18

What is g ?

- Force on body close to $r_E =$
 $GM_E m / r_E^2 = mg \rightarrow g = GM_E / r_E^2 = 9.81 \text{ m/s}^2$
- Constant for bodies near surface
- Assumed gravitational effect of Earth can be thought of as acting at center (ultimately justified for $p = 2$)

SG Planet X has free-fall acceleration 8 m/s^2 at the surface. Planet Y has twice the mass and twice the radius of planet X. On Planet Y

1. $g = 2 \text{ m/s}^2$.
2. $g = 4 \text{ m/s}^2$.
3. $g = 8 \text{ m/s}^2$.
4. $g = 16 \text{ m/s}^2$.
5. $g = 32 \text{ m/s}^2$.

Motivations for law of gravity

- Newton reasoned that Moon was accelerating – so a force must act
- Assumed that force was same as that which caused ‘apple to fall’
- Assume this varies like r^p
- Compare acceleration with known acceleration of Moon \rightarrow find p

Physics 215 – Fall 2019

Lecture 12-2 21

Apple and Moon calculation

$$a_M =$$

$$a_{\text{apple}} =$$

$$a_M/a_{\text{apple}} =$$

$$\text{But: } a_{\text{rad}} = \quad \omega =$$

$$a_M =$$

$$a_{\text{apple}} =$$

$$a_M/a_{\text{apple}} = 2.8 \times 10^{-4}$$

$$R_M/r_E =$$

$$\rightarrow p =$$

Physics 215 – Fall 2019

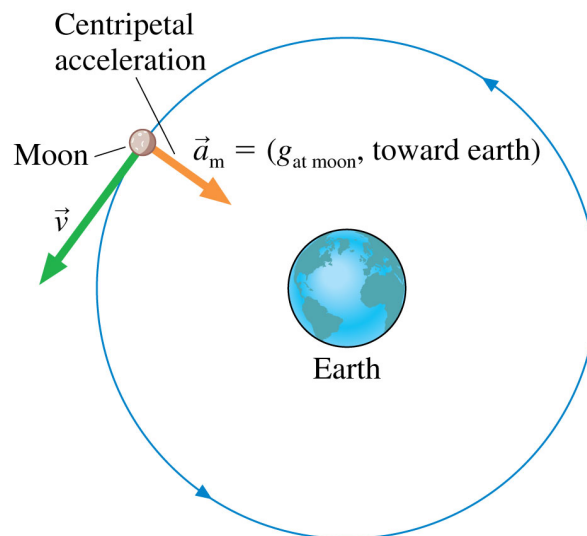
Lecture 12-2 22

SG The gravitational force between two asteroids is 1,000,000 N. What will the force be if the distance between the asteroids is doubled?

1. 250,000 N.
2. 500,000 N.
3. 1,000,000 N.
4. 2,000,000 N.
5. 4,000,000 N.



The moon is in free fall around the earth.



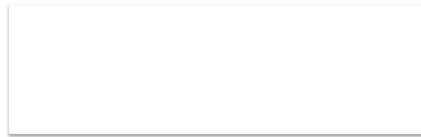
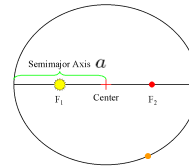
Kepler's Laws

experimental observations

3. Square of orbital period is proportional to cube of semimajor axis.

$$T^2 \sim a^3$$

- We can deduce this (for **circular orbit**) from gravitational law
- assume gravity responsible for acceleration in orbit →

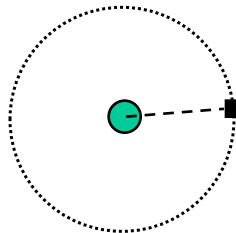


Orbits of Satellites

- Following similar reasoning to Kepler's 3rd law →

$$GM_E M_{\text{sat}} / r^2 = M_{\text{sat}} v^2 / r$$

$$v = (GM_E / r)^{1/2}$$



Gravitational Field

- Newton never believed in ***action at a distance***
- Physicists circumvented this problem by using new approach – imagine that every mass creates a ***gravitational field*** Γ at every point in space around it
- Field tells the magnitude (and direction) of the gravitational force on some test mass placed at that position $\rightarrow \mathbf{F} = m_{\text{test}}\Gamma$
- Close to earth: $\Gamma =$

Physics 215 – Fall 2019

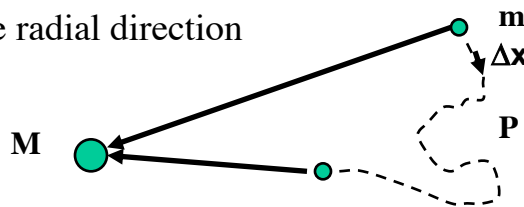
Lecture 12-2 27

Gravitational Potential Energy

Work done moving small mass along path P

$$W = \sum \mathbf{F} \cdot \Delta \mathbf{x}$$

But \mathbf{F} acts along the radial direction



Therefore, only component of \mathbf{F} to do work is along r

$$W = - \sum F(r) \Delta r$$

Independent of P!

Physics 215 – Fall 2019

Lecture 12-2 28

Gravitational Potential Energy

Define the gravitational potential energy $U(r)$ of some mass m in the field of another M as the work done moving the mass m in from infinity to r

$$\rightarrow U = \int_{\infty}^r F(r) \Delta r = -GMm/r$$

SG A football is dropped from a height of 2 m. Does the football's gravitational potential energy increase or decrease ?

1. decreases
2. increases
3. stays the same
4. depends on the mass of football

Gravitational Potential Energy near Earth's surface

$$U_h =$$

For small h/R_E :

$$\Delta U = (GM_E m/R_E^2)h = mgh!! \text{ as we expect}$$

Called the “flat earth approximation”

Recall: Energy conservation

- Consider mass moving in gravitational field of much larger mass M
- Energy conservation $\Delta E = 0$,
where $E = K+U = 1/2mv^2 - GmM/r$
- Hence work done by gravitational field changes kinetic energy: $W = -\Delta U = \Delta K$
- Notice $E < 0$ if object **bound**

Escape speed

- Object can just escape to infinite r if $E=0$

$$\rightarrow (1/2)mv_{\text{esc}}^2 = GM_E m/R_E$$

$$\rightarrow v_{\text{esc}}^2 = 2GM_E/R_E$$

- Magnitude ? 1.1×10^4 m/s on Earth
- What about on the moon ? Sun ?