Welcome back to Physics 215

Today's agenda:

- *Angular momentum*
- *Rolling without slipping*

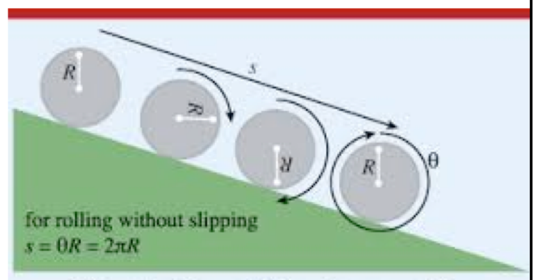
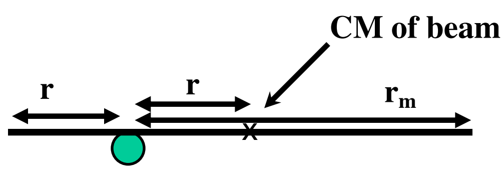


Figure 1. The conditions for pure rolling.

recall: extended free body diagram



Vertical equilibrium? $\sum F =$

Rotational equilibrium? $\sum \tau =$

Suppose M replaced by M/2 ?

- vertical equilibrium? $\sum F =$
- rotational dynamics?

$\sum \tau =$

- net torque?
- which way rotates?
- initial angular acceleration?

Rotational Kinetic Energy

$K = \sum_i (1/2) m_i v_i^2 =$

- Hence

$K =$

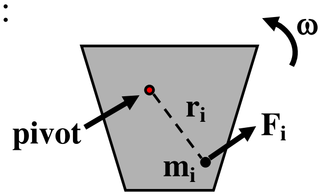
- This is the energy that a rigid body possesses by virtue of rotation

Angular Momentum

- can define rotational analog of linear momentum called ***angular momentum***
- in absence of ***external torque*** it will be conserved in time
- True even in situations where Newton's laws fail ….

Definition of Angular Momentum

* Back to slide on rotational dynamics:



* Rewrite, using $l_i = m_i r_i^2 \omega$:

* Summing over all particles in body:

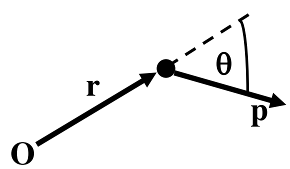
$$\Delta L/\Delta t = \tau_{ext}$$

L = angular momentum = $I\omega$

SG: An ice skater spins about a vertical exis through her body with her arms held out. As she draws her arms in, her angular velocity

A. increases
B. decreases
C. remains the same
D. need more information

Angular Momentum 1.



Point particle:

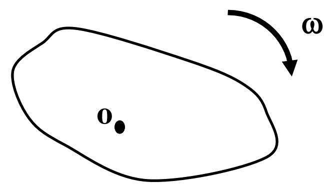
$|L| = |r||p|\sin(\theta) = m|r||v| \sin(\theta)$

vector form → **L** = **r** x **p**

– direction of L given by right hand rule (into paper here)

L = mvr if v is at 90^0 to r for single particle

Angular Momentum 2.



rigid body:

* $|L| = I\omega$ (fixed axis of rotation/fixed axle)
* direction – along axis – into paper here

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Rotational Dynamics

$$\tau = I\alpha$$

$$\Delta \mathbf{L}/\Delta t = \tau$$

- These are equivalent statements
- If no net external torque: $\tau = 0 \rightarrow$
  * **L** is constant in time
  * *Conservation of Angular Momentum*
  * Internal forces/torques do not contribute to external torque.

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Demo - wheels

Linear and rotational motion

- Force
- Acceleration

$$\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$$

- Momentum

$$\vec{p} = m\vec{v}$$

- Kinetic energy

$$K = \tfrac{1}{2} m v^2$$

- Torque
- Angular acceleration

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = I\vec{\alpha}$$

- Angular momentum**

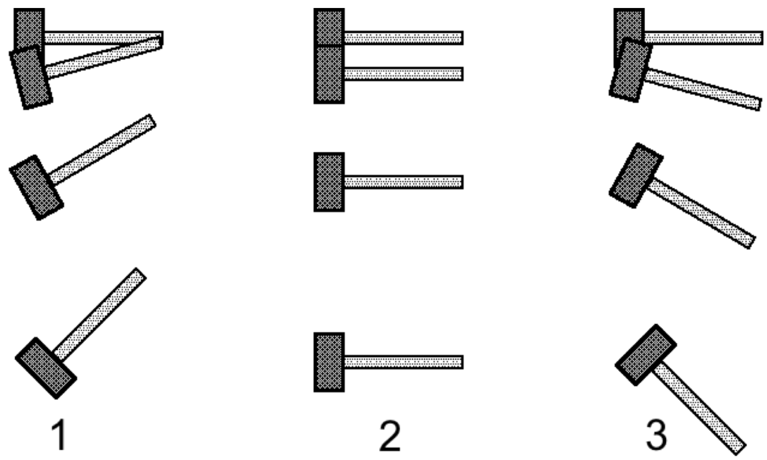
$$\vec{L} = I\vec{\omega}$$

- Kinetic energy

$$K = \tfrac{1}{2} I\omega^2$$

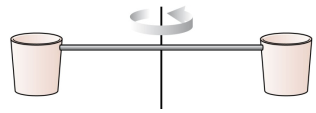
**** about a fixed axle or axis of symmetry**

SG A hammer is held horizontally and then released. Which way will it fall?



Falling bodies rotate about their center of mass!

SG Two buckets spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result,



A. The buckets speed up because the potential energy of the rain is transformed into kinetic energy.
B. The buckets continue to rotate at constant angular velocity because the rain is falling vertically while the buckets move in a horizontal plane.
C. The buckets slow down because the angular momentum of the bucket + rain system is conserved.
D. The buckets continue to rotate at constant angular velocity because the total mechanical energy of the bucket + rain system is conserved.
E. None of the above.

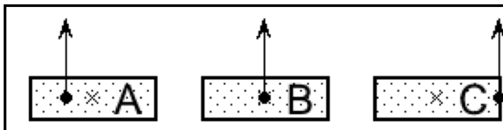
General motion of extended objects

- Net force ➔ acceleration of CM
- Net torque about CM ➔ angular acceleration (rotation) about CM
- Resultant motion is superposition of these two motions
- Total kinetic energy $K = K_{CM} + K_{rot}$

SG Three identical rectangular blocks are at rest on a flat, frictionless table. The same force is exerted on each of the three blocks for a very short time interval. The force is exerted at a different point on each block, as shown.

After the force has stopped acting on each block, which block will spin the fastest?

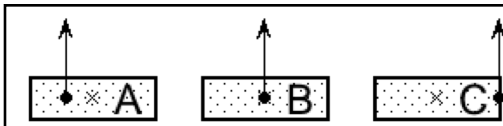
1. A.
2. B.
3. C.
4. A and C.



Top-view diagram

SG Three identical rectangular blocks are at rest on a flat, frictionless table. The same force is exerted on each of the three blocks for a very short time interval. The force is exerted at a different point on each block, as shown.

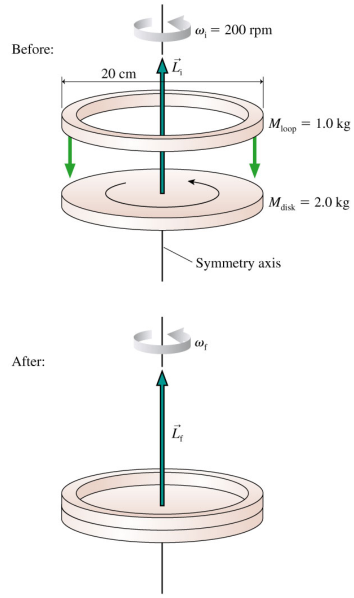
After each force has stopped acting, which block's center of mass will have the greatest speed?



Top-view diagram

1. A.
2. B.
3. C.
4. A, B, and C have the same C.O.M. speed.

A 20-cm diameter, 2.0 kg solid disk is rotating at 200 rpm. A 20-cm-diameter, 1.0 kg circular loop is dropped straight down onto the rotating disk. Friction causes the loop to accelerate until it is riding on the disk. What is the final angular velocity of the combined system?



Rolling without slipping

$\Delta x_{cm} =$

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$v_{cm} =$

$v_{cm} =$

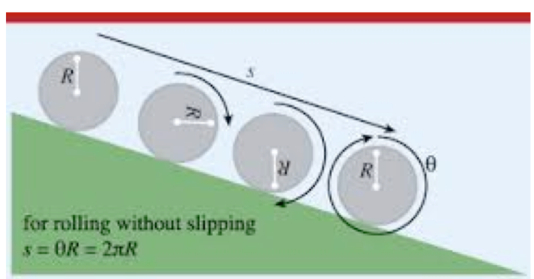
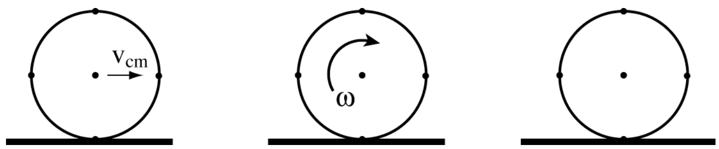


Figure 1. The conditions for pure rolling.

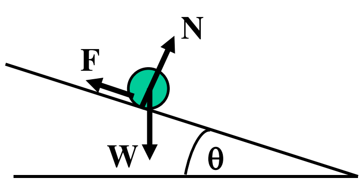
Rolling without slipping



v_{cm} =

a_{cm} =

Rolling without slipping



$\Sigma F = ma_{CM}$

$\Sigma \tau = I\alpha$

Now $a_{CM} = R\alpha$ if no slipping

So, ma_{CM}

and F =

Kinetic energy of rolling

- The total kinetic energy of a rolling object is the sum of its rotational and translational kinetic energies:

SG. Two cylinders with the same radius and same total mass roll down a ramp. In cylinder A, a set of 8 point masses are equally spaced in a circle with radius r1 around the cylinder's axis of rotation, while in cylinder B, the 8 point masses are a distance r2 > r1 from the center. Which cylinder reaches the bottom of the ramp first?

A. Cylinder A
B. Cylinder B
C. They both reach the bottom at the same time
D. Not enough information to tell