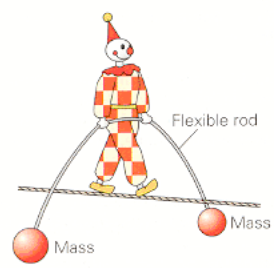
Welcome back to Physics 215

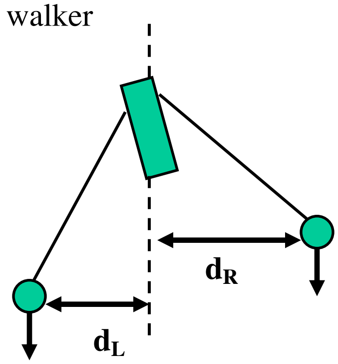
Today's agenda:

- *Torque*
- *Rotational Dynamics*
- *Moment of Inertia*



Recall: Restoring torque

e.g., wire walker



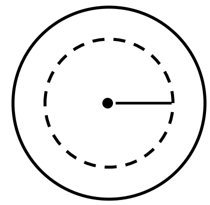
Consider displacing anticlockwise

- τ_R increases
- τ_L decreases

net torque causes clockwise rotation!

Recall:Rotations about fixed axis

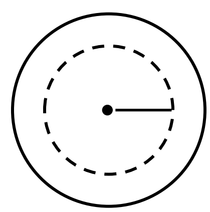
- Linear speed: $v = (2\pi r)/T = \omega r$. Quantity ω is called ***angular velocity***



- $\boldsymbol{\omega}$ is a vector! Use right hand rule to find direction of $\boldsymbol{\omega}$.
- Angular acceleration $\boldsymbol{\alpha} = \Delta\boldsymbol{\omega}/\Delta t$ is also a vector!
  - $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ *parallel* $\rightarrow$ angular speed increasing
  - $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ *antiparallel* $\rightarrow$ angular speed decreasing

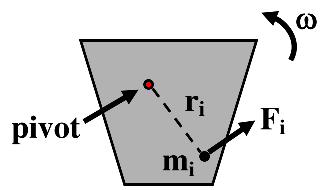
Relating linear and angular kinematics

- Linear speed: $v = (2\pi r)/T = \omega r$



- Tangential acceleration: $a_{tan} = r\alpha$
- Radial acceleration: $a_{rad} = v^2/r = \omega^2 r$

Rotational Motion



* Particle *i*:

$|v_i| = r_i\,\omega$ at 90° to r_i

* Newton's 2nd law:

$m_i \Delta v_i/\Delta t = F_i^T$ ← component at 90° to r_i

* Substitute for v_i and multiply by r_i :

$m_i r_i^2 \Delta\omega/\Delta t = F_i^T r_i = \tau_i$

* Finally, sum over all masses:

$$(\Delta\omega/\Delta t) \sum m_i r_i^2 = \sum \tau_i = \tau_{net}$$

Discussion

$$(\Delta\omega/\Delta t) \sum m_i r_i^2 = \tau_{net}$$

α – angular acceleration

Moment of inertia, I

$$I\alpha = \tau_{net}$$

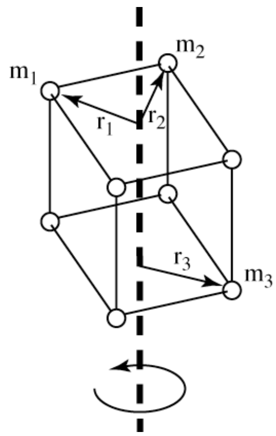
compare this with Newton's 2nd law

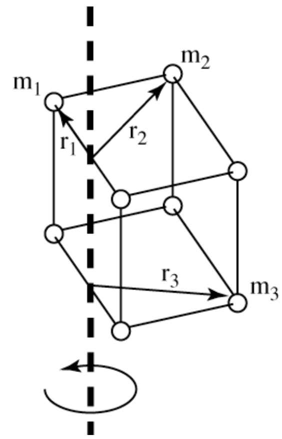
$$Ma = F$$

Moment of Inertia

$$\boxed{I = m_1 r_1^2 + m_2 r_2^2 + \ldots + m_N r_N^2 = \sum_{i=1}^{N} m_i r_i^2}$$

* I must be defined with respect to a particular axis

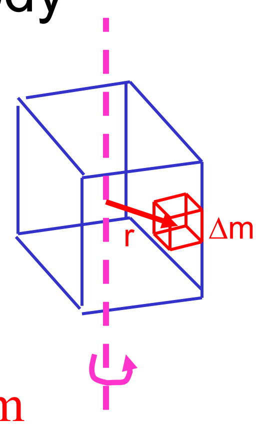




Moment of Inertia of Continuous Body

$\Delta m \mapsto 0$

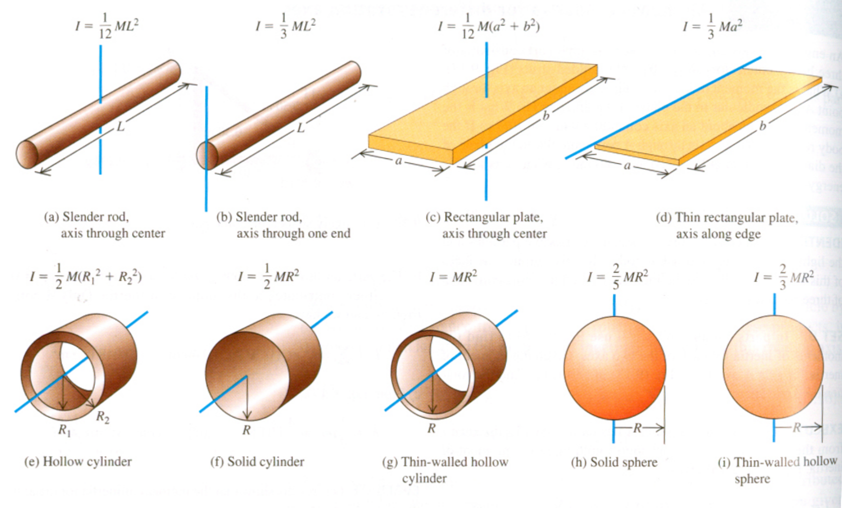
$$\sum \Rightarrow \int$$



$$I = \sum_{i=1}^{N} m_i r_i^2 \Rightarrow I = \int r^2 dm$$

Tabulated Results for Moments of Inertia of some rigid, uniform objects

Table 9.2 Moments of Inertia of Various Bodies



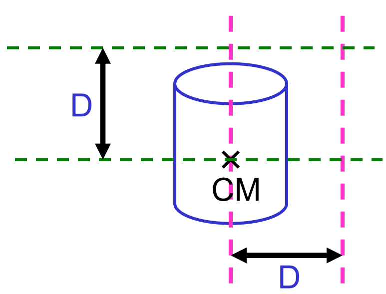
(from p.299 of *University Physics*, Young & Freedman)

Demo - wheels

SG Two identical masses are equally spaced from the center of a rod. For rod A, the masses are 15 cm apart. For rod B, the masses are 75 cm apart. A hand applies the same torque to the rods about their center of mass. Which rod has the larger angular acceleration?

A. Rod A
B. Rod B
C. They are the same
D. Can't tell from the information given.

Parallel-Axis Theorem



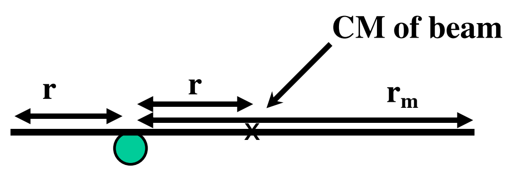
$$\boxed{I = I_{CM} + M D^2}$$

*Smallest I will always be along axis passing through CM

Practical Comments on Calculation of Moment of Inertia for Complex Object

1. To find I for a ***complex*** object, **split** it into ***simple*** geometrical shapes that can be found in Table 9.2
2. Use Table 9.2 to get I_{CM} for each part about the axis ***parallel*** to the axis of rotation and ***going through the center-of-mass***
3. If needed use **parallel-axis theorem** to get I for each part about the axis of rotation
4. **Add** up moments of inertia of all parts

recall: extended free body diagram



Vertical equilibrium? $\sum F =$

Rotational equilibrium? $\sum \tau =$

Suppose M replaced by M/2 ?

- vertical equilibrium? $\sum F =$
- rotational dynamics?

$\sum \tau =$

- net torque?
- which way rotates?
- initial angular acceleration?

Rotational Kinetic Energy

$K = \sum_i (1/2) m_i v_i^2 =$

- Hence

$K =$

- This is the energy that a rigid body possesses by virtue of rotation

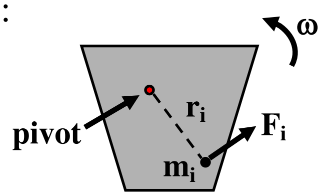
Angular Momentum

- can define rotational analog of linear momentum called ***angular momentum***
- in absence of ***external torque*** it will be conserved in time
- True even in situations where Newton's laws fail ….

Definition of Angular Momentum

* Back to slide on rotational dynamics:

$m_i r_i^2 \Delta\omega/\Delta t = \tau_i$



* Rewrite, using $l_i = m_i r_i^2 \omega$:

$\Delta l_i/\Delta t = \tau_i$

* Summing over all particles in body:

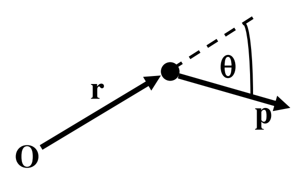
$$\Delta L/\Delta t = \tau_{ext}$$

L = angular momentum = $I\omega$

SG: An ice skater spins about a vertical exis through her body with her arms held out. As she draws her arms in, her angular velocity

A. increases
B. decreases
C. remains the same
D. need more information

Angular Momentum 1.



Point particle:

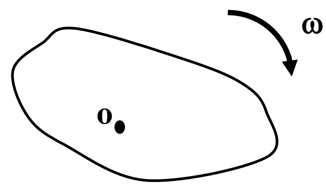
$|L| = |r||p|\sin(\theta) = m|r||v|\sin(\theta)$

vector form → **L** = **r** x **p**

– direction of L given by right hand rule (into paper here)

L = mvr if v is at 90^0 to r for single particle

Angular Momentum 2.



rigid body:

* |L| = $I\omega$ (fixed axis of rotation/fixed axle)
* direction – along axis – into paper here

Rotational Dynamics

$$\tau = I\alpha$$

$$\Delta \mathbf{L}/\Delta t = \tau$$

- These are equivalent statements
- If no net external torque: $\tau = 0$ →
  * **L** is constant in time
  * *Conservation of Angular Momentum*
  * Internal forces/torques do not contribute to external torque.

Demo - wheels

Linear and rotational motion

- Force
- Acceleration

$$\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$$

- Momentum

$$\vec{p} = m\vec{v}$$

- Kinetic energy

$$K = \tfrac{1}{2} m v^2$$

- Torque
- Angular acceleration

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = I\vec{\alpha}$$

- Angular momentum**

$$\vec{L} = I\vec{\omega}$$

- Kinetic energy

$$K = \tfrac{1}{2} I\omega^2$$

**** about a fixed axle or axis of symmetry**

SG A hammer is held horizontally and then released. Which way will it fall?