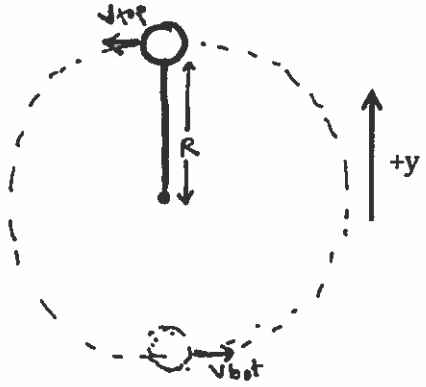
Name: Solutions

1. [25 pts] A yo-yo is being swung in a vertical circle, as shown in the diagram to the right, where acceleration due to gravity points downward. The radius of the circle is R = 0.5 m.



   a. [4 pts]At the top of the circle, the yo-yo is moving at a velocity of 4 m/s. Draw a free body diagram for the yo-yo at the top of the circle.

   b. [5pts] What is the tension in the string at the top of the circle?

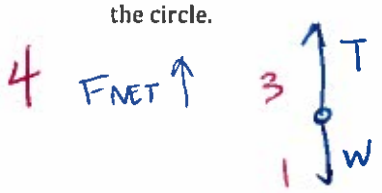
$F_{NET} = \frac{mv^2}{R}$ (2)

$T + W_y = \frac{mv^2}{R} \Rightarrow T = \frac{mv^2}{R} - mg$ (2)

$= m\left(\frac{v^2}{R} - g\right)$

$= (22.2\ m/s^2)\,m$

   c. [4pts] At the bottom of the circle, the yo-yo is moving at a velocity of 6 m/s. Draw a free body diagram at the bottom of the circle.



$F_{NET} = \frac{mv^2}{R} = T - W$

$\Rightarrow T = m\left(\frac{v^2}{R} + g\right) = \left(81.8\ \frac{m}{s^2}\right)m$

   d. [5pts] What is the tension in the string at the bottom of the circle?

   e. [4pts] What would the tension of the string be at the bottom of the circle if the yo-yo trick was being performed in an elevator accelerating downward at 2 m/s^2?

$T - W = ma_{tot} = m\frac{v^2}{R} - ma_e \Rightarrow T = \frac{mv^2}{R} - ma_e + mg$ (2)

$T = m\left(\frac{v^2}{R} - a_e + g\right)$

$= 79.8\ m/s^2\ m$ (1)

   f. [3 pts] What would the tension of the string be at the bottom of the circle if the yo-yo trick was being performed in an elevator moving upward at a constant velocity of 5 m/s?

Same as d $(81.8\ m/s^2)\,m$

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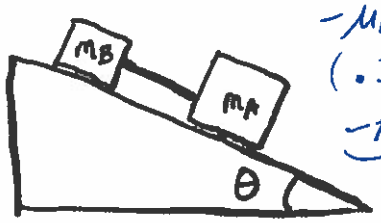
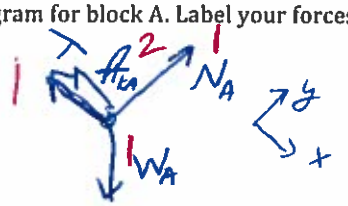
2. [30 pts] Two blocks are connected by a string as shown in the diagram on the right. Here m_A = 4.00 kg, m_B = 2.00 kg, μ_{kA} = 0.300, and μ_{kB} = 0.400. The ramp angle θ is 30 degrees.

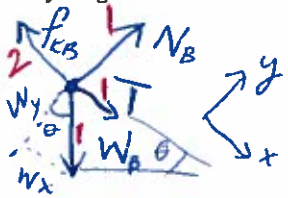


$-\mu_{KA} m_A - \mu_{KB} m_B$
$(.3)\,4 \quad (.4)(2)$ kg
$-1.2 - 0.8$
-2.0

a. [5 pts] Draw a free body diagram for block A. Label your forces.



b. [5pts] Draw a free body diagram for block B. Label your forces.



c. [5 pts] Which block experiences a larger net force? Explain your answer.

$a_A = a_B$ $\quad m_A > m_B$ $\quad \therefore F_{NET_A} = m_A a_A > m_B a_B = F_{NET_B}$

$F_{NET_A} > F_{NET_B}$.

d. [8 pts] Find the acceleration of the system down the plane.

$W_x = mg\sin\theta$ } for both blocks.
$W_y = mg\cos\theta$ }

$N_A = m_A g\cos\theta$

$-\mu_{KA} m_A g\cos\theta - T + m_A g\sin\theta = m_A a_x$

$-\mu_{KB} m_B g\cos\theta + T + m_B g\sin\theta = m_B a_x$

$(-\mu_{KA} m_A - \mu_{KB} m_B) g\cos\theta + (m_A + m_B) g\sin\theta = (m_A + m_B) a_x$

$$a_x = \frac{(-\mu_{KA} m_A - \mu_{KB} m_B) g\cos\theta + (m_A + m_B) g\sin\theta}{(m_A + m_B)}$$

$$= \frac{[(-2.0)[0.866] + [6][0.5]]g}{6}$$

$$= \frac{-1.73 + 3}{6} g = \boxed{2.07\ m/s^2}$$

e. [7 pts] Find the tension in the connecting string.

$-\mu_{KA} m_A g\cos\theta + m_A g\sin\theta - m_A a_x = T$

$m_A g(-\mu_{KA}\cos\theta + \sin\theta - 0.21) = T$

$-0.26 + 0.5 - 0.21$

$(4\,kg)(9.8\ m/s^2)(0.03) = T$

$T = 1.14\,N$

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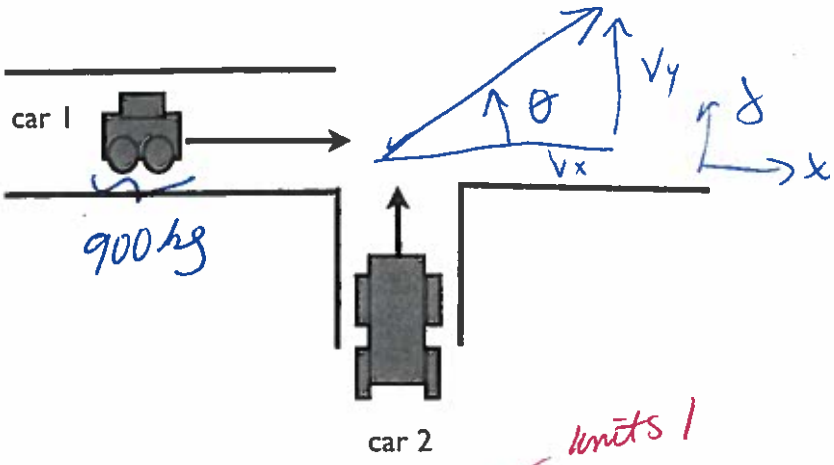
4. [25 pts] Car 1 of mass 900 kg traveling east at 22 m/s collides with a Car 2 of mass 1200 kg which is heading north at 28 m/s as shown in the diagram. After the collision the cars stick together to form a single wreck. Denote the easterly direction as the x axis and north as the y direction.



$m_A + m_B = 2100\ kg$

a) (7 pts) What are the x and y components of the total momentum of the system before the collision?

units !

$p_{ix} = 900\ kg \cdot 22\ m/s = 19800\ kg \cdot m/s\ \hat{i}$

$p_{iy} = 1200\ kg\ 28\ m/s = 33600\ kg \cdot m/s\ \hat{j}$

b) (8 pts) What will be the total x and y components of momentum after the collision? Calculate the speed of the combined wreck after the collision. At what angle (in degrees) relative to the x axis does the wreck move after the collision?

2 $\vec{P}_f = 19{,}800\ kg \cdot m/s\ \hat{i} + 33{,}600\ kg\ m/s\ \hat{j} = (m_A + m_B)\ \vec{v}_f$

2 $v_f = 9.43\ m/s\ \hat{i} + 16\ m/s\ \hat{j}$ $\Rightarrow 2\ |v_f| = \sqrt{89 + 256} = 18.57\ m/s$

2 $\tan\theta = \frac{v_y}{v_x} \Rightarrow \theta = 59.5^\circ$

c) (5 pts) During the collision car 1 exerts an average force on car 2 whose magnitude F12 is 15000 N. What is the magnitude of the force exerted by car 2 on car 1? Describe the relative directions of these two forces.

$|F_{21}| = |F_{12}| = 15000N$, act in opposite directions

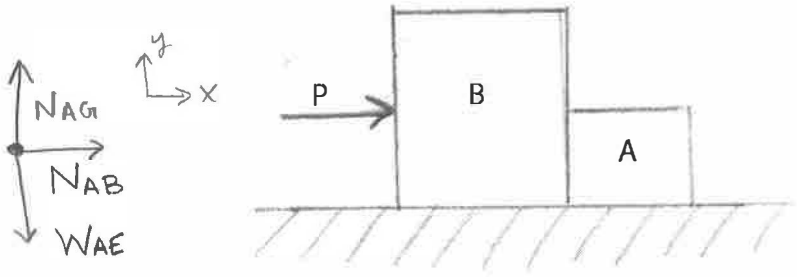
d) (5 pts) If these forces act for approximately 0.5 secs calculate the impulse associated with F12

$I = F\Delta t$

$I = 7{,}500\ N \cdot s$

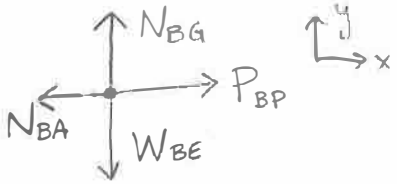
1. [25 pts] A truck flipped over and spilled its contents onto a frozen lake. A member of the clean-up crew is pushing two boxes, which are in contact, across the frictionless, flat surface of the frozen lake, as shown in the figure. The boxes accelerate to the right. Box A has a smaller mass than box B. *You must show all of your work to receive full credit.*

a) [5 pts] Draw the free body diagram for box A. Label your forces with 2 subscripts indicating which body the force acts on and which body is supplying the force.



N_{AG} = Normal force on A due to G

b) [5 pts] Draw a free body diagram for box B. Label your forces as in part a.



c) [4 pts] Is the normal force on box A due to the ground greater than, smaller than, or equal to the normal force on box B due to the ground? Explain your answer.

$a_y = 0 \rightarrow |N_{BG}| = |W_{BE}| + |N_{AG}| = |W_{AE}|$

but $m_B > m_A \Rightarrow |W_{AE}| < |W_{BE}| \Rightarrow |N_{AG}| < |N_{BG}| \Rightarrow$ smaller than

d) [4 pts] Which box experiences the larger net force? What is the direction of this force? Explain your answer.

$a_{BX} = a_{AX}$
$m_B > m_A$
$m_B a_{BX} > m_A a_{AX}$

$F_{net\,x} = ma_x \Rightarrow F_{net\,BX} = m_B a_{BX}$
$F_{net\,y} = 0 \qquad F_{net\,AX} = m_A a_{AX} \Rightarrow$

$F_{net\,BX} > F_{net\,AX} \Rightarrow$ B experiences the larger net force.

e) [2pts]Identify which forces are Newton third law pairs.

N_{AB}, N_{BA} only.

f) [5 pts] If the mass of box A is 1/4 that of box B, the boxes accelerate at a rate a_1. If the mass of box A is ½ that of box B, what is the acceleration a_2 of the boxes? (Write a_2 in terms of a_1.)

for both objects, $F_{net\,x} = (m_B + m_A)a_x \Rightarrow a_{1x} = \frac{P}{m_B + m_A} \Rightarrow a_{1x} = \frac{P}{\frac{5}{4}m_B} = \frac{4}{5}\frac{P}{m_B} \Rightarrow \frac{5}{4}a_{1x} = \frac{P}{m_B}$ $\quad m_A = \frac{1}{4}m_B$

$a_{2x} = \frac{P}{m_B + m_A} \Rightarrow \frac{P}{\frac{3}{2}m_B} = \frac{2}{3}\frac{P}{m_B} = \frac{2}{3}\left(\frac{5}{4}a_{1x}\right)$

$\boxed{a_{2x} = \frac{5}{6}a_{1x}}$