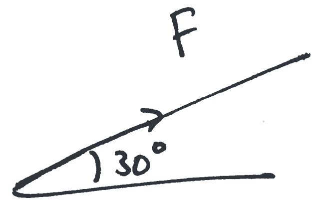
Homework (Week-8) Solutions.

Ch : 7

Problem 28:

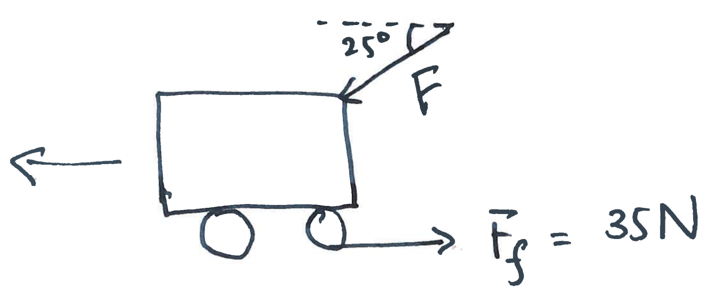


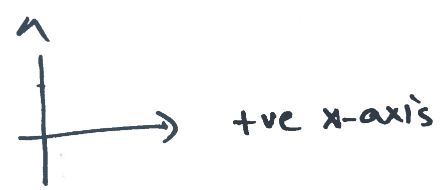
$$W = \vec{F}\cdot\vec{x} \qquad |\vec{x}| = 30\text{m}.$$

$$= |\vec{F}||\vec{x}|\cos\theta$$

$$= (50)(30)\cos(30^\circ) = 1299.038\text{ J}.$$

Problem 29:





(a) Work done by friction $= (35)(-20) = -700$ J.

(b) Work done by gravitational force is zero since displacement is $\perp$ to direction of gravity.

(c) Work done by shopper $= (-F\cos 25^\circ)(-20)$

$= 20F\cos 25^\circ$

(d) Using "work-done by net force equals change in kinetic energy" we have $20F\cos 25^\circ = 700$.

$F = 38.61$ N

Problem 48:

$$K.E = \frac{1}{2}mv^2$$

$$\frac{1}{2}(5)v_1^2 = 3\left(\frac{1}{2}(8)v_2^2\right).$$

$$\frac{5}{2}v_1^2 = 12v_2^2$$

$$\frac{v_1^2}{v_2^2} = \frac{24}{5}.$$

$$\frac{v_1}{v_2} = \sqrt{\frac{24}{5}}.$$

Problem 58:

Since there are no dissipative forces.

$$E_i = E_f.$$

E ≡ Total mechanical energy.

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f.$$

$$\left.\begin{array}{l} y_i = 100m \quad v_i = 0 \\ y_f = 0 \end{array}\right\} \Rightarrow \frac{1}{2}\cancel{m}v_f^2 = \cancel{m}gy_i$$

$$v_f = \sqrt{2gy_i}$$
$$= \sqrt{2(10)(100)}$$
$$= 44.72 \text{ m/s}.$$

Ch-9:

Problem 30:

(a) $a = \dfrac{v_f^2 - v_i^2}{2(\Delta s)} = \dfrac{0^2 - (7)^2}{2(0.01)}$

$= \dfrac{-49}{2(0.01)}$

$= -2450\ m/s^2.$

$0.01 = 7(\Delta t) - \frac{1}{2}(2450)(\Delta t)^2.$

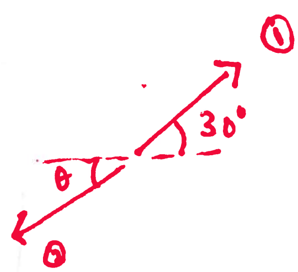
$\Delta t = 0.002857$ sec.

(b) $F_{ave} = \dfrac{\Delta p}{\Delta t} = \dfrac{(0.45)(7)}{0.002857} = 1102.55\,N.$

Problem 36:

m_1 ① $\rightarrow$ 6 m/s $= v_1$ m_2 ② $v_2 = 0.$

① $\longrightarrow$ ② $\Rightarrow$. Assume



Here puck 1 has an incoming speed of 6 m/s while puck 2 is at rest.

$\vec{P} \equiv$ total momentum

$\left.\begin{array}{l}\vec{P} = \vec{P_1} + \vec{P_2} \\ \vec{P}_{initial} = m_1 6\,\hat{i}.\end{array}\right\}$ before collision

$\vec{P}_{final} = \vec{P_1}' + \vec{P_2}'$

$\vec{P_1}' = (v_1'\cos 30°\,\hat{i} + v_1'\sin 30\,\hat{j})$

$P_2' = (-v_2'\cos\theta\,\hat{i} - v_2'\sin\theta\,\hat{j})m$

By conservation of momentum.

$$\vec{P}_{initial} = \vec{P}_{final}$$

$$\Rightarrow \cancel{m_1}(V_1' \cos 30^\circ - V_2' \cos\theta) = 6 \cancel{m_1}$$

plug this in here.

$$m_2(V_1' \sin 30^\circ - V_2' \sin\theta) = 0.$$

$$\Rightarrow \cancel{V_2} = \cancel{\frac{V_1' \sin 30^\circ}{\sin\theta}}.$$

$$V_2' = \frac{V_1' \sin 30^\circ}{\sin\theta}$$

By conservation of energy.

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2.$$

$$V_2 = 0.$$

$$\frac{1}{2} m_1 (6)^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2.$$

Plucks. are identical $\Rightarrow$ $m_1 = m_2$.

$$\frac{(6)^2}{2} = v_1'^2 + v_2'^2.$$

$$v_1'^2 + v_2'^2 = 18.$$

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since $m_1 = m_2$ we have. three equations and three unknowns

$$v_1' \cos 30^\circ - v_2' \cos\theta = 6.$$

$$v_1'^2 + v_2'^2 = 18.$$

$$V_2' = \frac{v_1' \sin 30^\circ}{\sin\theta}$$

$$v_1' \cos 30^\circ - v_1' \sin 30^\circ \cdot \frac{\cos\theta}{\sin\theta} = 6 \Rightarrow v_1' = \frac{6}{\cos 30^\circ + \sin 30^\circ \tan}$$

Similarly.

$$v_1'^2 + \left(\frac{v_1' \sin 30}{\sin\theta}\right)^2 = 18.$$

$$v_1'^2 \left(1 + \frac{\sin^2(30)}{\sin^2\theta}\right) = 18.$$

$$\left(\frac{6}{\cos 30^\circ + \sin 30^\circ \tan\theta}\right)^2 \left(1 + \frac{\sin^2(30)}{\sin^2\theta}\right) = 18.$$

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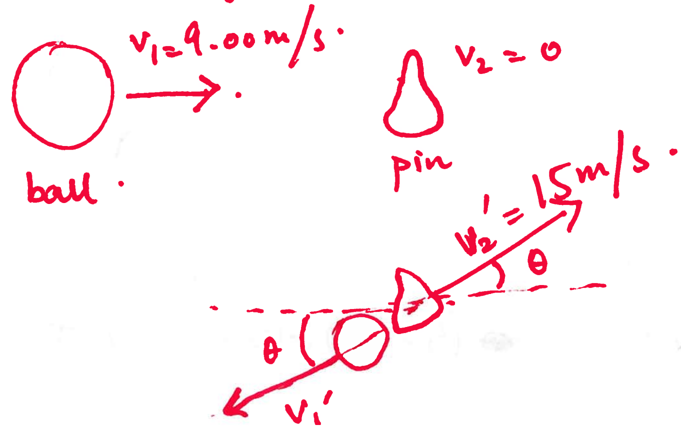
solve for θ the equation given above and plug it back to find v_1' and then use v_1' to find v_2'

Problem 43:

Before collision.



Using conservation of momentum.

$$m_1 V_{1x} + m_2 V_{2x} = m_1 V_{1x}' + m_2 V_{2x}'$$

$$m_1 V_{1y} + m_2 V_{2y} = m_1 V_{1y}' + m_2 V_{2y}'$$

$V_{2x} = V_{2y} = 0.$

$V_{1y} = 0.$

$$9m_1 = m_1 V_{1x}' + m_2 V_{2x}' \quad \text{— (i)}$$

$$0 = m_1 V_{1y}' + m_2 V_{2y}' \quad \text{— (ii)}.$$

$V_{1x}' = -15\cos\theta.$

$V_{1y}' = -15\sin\theta.$

$V_{2x}' = +V_2' \cos\theta$

$V_{2y}' = +V_2' \sin\theta.$

$\Rightarrow$ Using eqn - (ii)

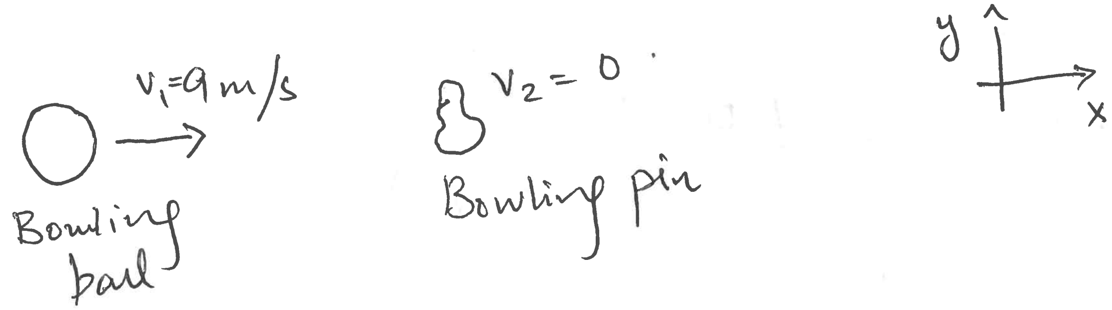
$$-(5.50)(15\sin\theta) + 0.85\, V_2' \sin\theta = 0.$$

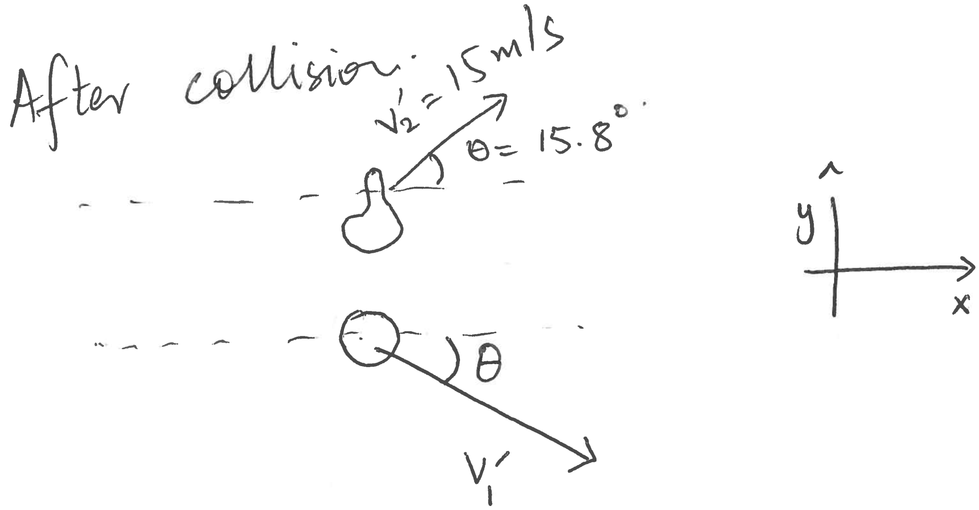
$$\Rightarrow V_2' = \frac{(5.50)(15)}{0.85} = 97.05\,\text{m/s}$$

using eqn (i)

$$9(5.50) = (5.50)(-15\cos\theta) + (0.85)(97.05)\cos\theta$$

Problem 43:





Using momentum conservation in the direction orthogonal to direction of impact.

$$m_2 v_2' \sin(15.8^\circ) - m_1 v_1' \sin\theta = m_1 v_{1y} + m_2 v_{2y}$$

$$(0.85)(15)\sin(15.8) - 5.50\, v_1' \sin\theta = 0 \Rightarrow v_1' = \frac{3.471}{5.50\sin\theta}$$

Similarly in x-direction.

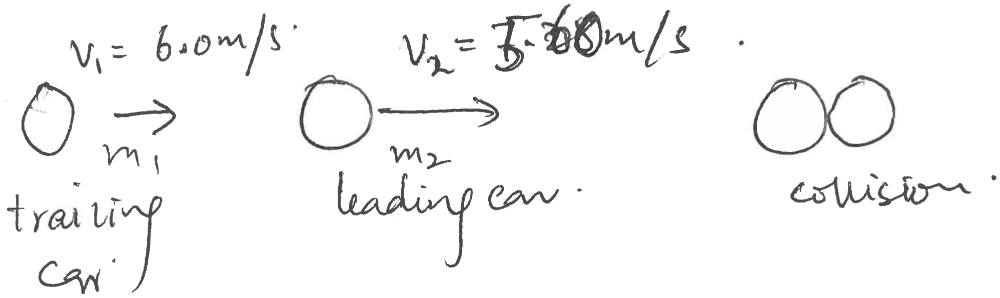
$$m_2 v_2' \cos(15.8^\circ) + m_1 v_1' \cos\theta = m_1 v_{1x} + m_2 v_{2x}$$

$$(0.85)(15)\cos(15.8) + \frac{(\cancel{5.50})(3.471)}{\cancel{5.50}}\left(\frac{\text{[redacted]}1}{\tan\theta}\right) = (9)(5.50)$$

$$\Rightarrow \theta = 5.326^\circ. \qquad v_1' = 6.798\,m/s.$$

Problem 48:

Identical cars with same mass.



$m_2 = 1.3 m_1$

$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$

$V_1 + V_2 (1.3) = V_1' + 1.3 V_2'$

$V_1 + (1.3) V_2 = V_1' + (1.3) V_2'$

$6 + (1.3)(5.60) = V_1' + 1.3 V_2'$ — (i)

By energy conservation.

$\frac{1}{2} m V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} m_1 (V_1')^2 + \frac{1}{2} m_2 (V_2')^2$

$V_1^2 + 1.3 V_2^2 = (V_1')^2 + 1.3 (V_2')^2$

$6^2 + (1.3)(5.60)^2 = (V_1')^2 + 1.3 (V_2')^2$ — (ii)

Use eqn (i) to find either v_1' or v_2' in terms of the other. Plug that expression into eqn (ii) and solve the quadratic eqn either in v_1' or v_2'.

Problem 51:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$= (m_1 + m_2) v'$$

since after collision sled and the 2nd child move together as one object

$$(35)(3.5) + 0 = (35 + 35) v'$$

$$v' = \frac{1}{2}(3.5)$$

$$v' = 1.75 \, m/s$$

Problem 55:

$V_{ix} = 50\cos(40^\circ).$

At highest point $V_{iy} = 0.$

(a) momentum in y-direction after collision (explosion in this case).

$m_3 V_{3,fy} + m_1 V_{1,fy} = 0.$ (since

$(0.3)(V_{3,f}) + (1.0)(-10) = 0.$

$$V_{3,f} = \frac{10}{0.3}$$

$$= 33.33\,m/s.$$

$m_2 V_{2,x} = m V_{ix}.$

$(0.7)(V_{2,x}) = (2)(50\cos(40^\circ))$

$V_{2,x} = 109.43\,m/s.$

(b) $\Delta y = (33.33)(\Delta t) - \frac{1}{2}(10)(\Delta t)^2$ where $\Delta t = \frac{33.33}{10}$

Δy = height above the break-up point.

(c) time taken by 0.7 kg piece of exploded mass to land is found by first finding the height at which the explosion happened.

$V_{iy} = 0$ at explosion point $\Rightarrow t = \dfrac{50 \sin(40^\circ)}{g}$

$$y = (50 \sin 40^\circ)\left(\frac{50 \sin 40^\circ}{g}\right) - \frac{1}{2}g \frac{(50 \sin 40^\circ)^2}{g^2}$$

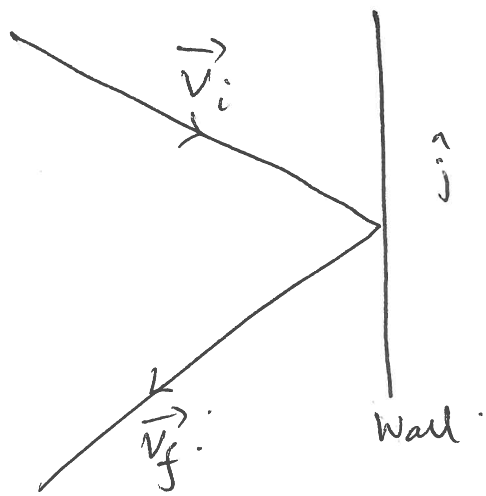
$$y = \frac{50^2 \sin^2(40^\circ)}{2g},$$

$$y = \frac{50^2 \sin^2(40)}{20},$$

time taken by m_2 to fall vertical distance y is given by $t = \sqrt{\dfrac{2y}{g}}$

Rest of the calculation is simple.

Problem 105:



If collision is elastic $\hat{i}$ component of velocity is reversed.

$$\vec{v}_f = (2.2\hat{i} - 0.4\hat{j})\ m/s.$$