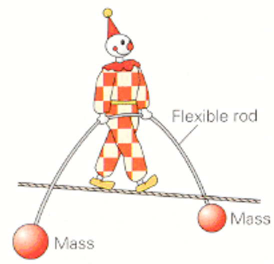
Welcome back to Physics 215

Today's agenda:

- *Torque*
- *Rotational Dynamics*
- *Moment of Inertia*



Situations with $F_{net} = 0$

- Consider 2 particles and $M = m_1+m_2$
- CM definition → $Mr_{CM} = m_1r_1 + m_2r_2$

$$M\Delta r_{CM}/\Delta t = m_1\Delta r_1/\Delta t + m_2\Delta r_2/\Delta t = m_1v_1 + m_2v_2$$

- RHS is total momentum

Thus, *velocity of center of mass is constant in absence of external forces!*

Conclusions

If there is ***no net force*** on a system,
the center of mass of the system:

- will stay at rest if it is initially at rest, or
- will continue to move with the same velocity if it is initially moving.

What about F_{ext} not zero?

Motion of center of mass of a system:

$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{CM}}$$

The center of mass of a system of point objects moves in the same way as a single object with the same total mass would move under the influence of the same net (external) force.

Throwing an extended object

- odd-shaped object – 1 point has simple motion = projectile motion
  - *center of mass*

- Total external force F_{ext}

$$a_{CM} = F_{ext} /M$$

- ***Translational*** motion of system looks like all mass is concentrated at CM

Equilibrium of extended object

- Clearly net force must be zero

- Also, if want object to behave as point at center of mass → **ALL forces acting on object must pass through CM**

Using the center of mass as the origin for a system of objects:

$$0 = \vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

Therefore, the center of mass is the point which, if taken as the origin, makes:

$$(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots) = 0$$

Restatement of equilibrium conditions

$m_1r_1 + m_2r_2 = 0 \rightarrow m_1gr_1 + m_2gr_2 = 0$

- Thus, Σ force x displacement = 0
- quantity – force x displacement called ***torque (preliminary definition)***

Thus, equilibrium requires net torque to be zero

Conditions for equilibrium of an extended object

For an extended object that remains at rest and does not rotate:

- The net force on the object has to be zero.

$$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}} = 0$$

- The net torque on the object has to be zero.

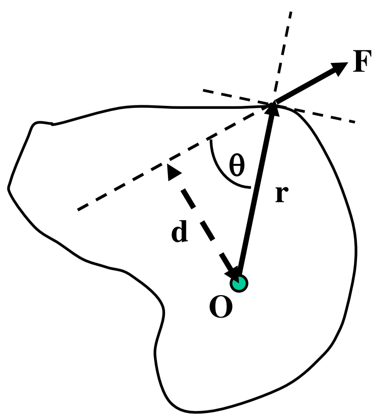
$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = 0$$

Preliminary definition of torque:

$$\tau = F\,d$$

The torque on an object with respect to a given pivot point and due to a given force is defined as the product of the force exerted on the object and the moment arm. The moment arm is the perpendicular distance from the pivot point to the line of action of the force.

Computing torque



$|\tau| = |F|d$
$= |F||r|\sin\theta$
$= (|F| \sin\theta)|r|$

component of force at 90^0 to position vector times distance

Definition of torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where ***r*** is the vector from the reference point (generally either the pivot point or the center of mass) to the point of application of the force ***F***.

$$|\vec{\tau}| = r\ F \sin\theta$$

where θ is the angle between the vectors ***r*** and ***F***.

Vector (or “cross”) product of vectors

The vector product is a way to combine two vectors to obtain a *third vector* that has some similarities with multiplying numbers. It is indicated by a cross (×) between the two vectors.

The ***magnitude*** of the vector cross product is given by:

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

The ***direction*** of the vector **A**×**B** is *perpendicular* to the plane of vectors **A** and **B** and given by the right-hand rule.

Interpretation of torque

- Measures tendency of any force to cause rotation
- Torque is defined with respect to some origin – must talk about “torque of force about point X”, etc.
- Torques can cause clockwise (-) or anticlockwise rotation (+) about pivot point

Extended objects need *extended free-body diagrams*

- Point free-body diagrams allow finding net force since points of application do not matter.

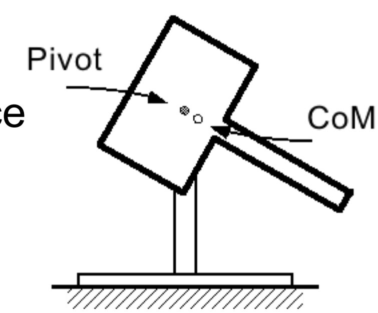
$$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}}$$

- *Extended free-body diagrams* show point of application for each force and allow finding net torque.

$$\vec{\tau}_{\text{net}} = \sum \vec{\tau}$$

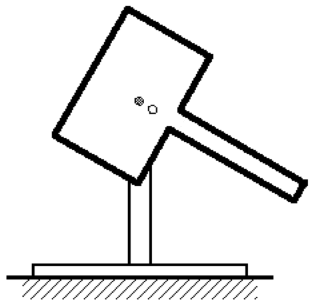
SG A T-shaped board is supported such that its center of mass is to the right of and below the pivot point.

Which way will it rotate once the support is removed?

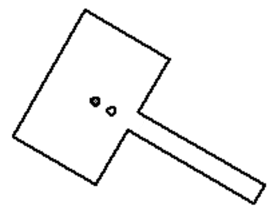


1. Clockwise.
2. Counter-clockwise.
3. Not at all.
4. Not sure what will happen.

Center of mass is *to the right of* and *below* pivot



Extended free-body diagram



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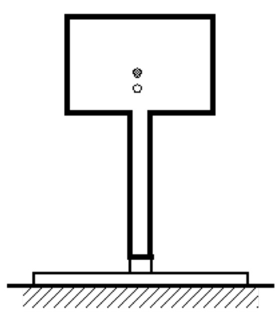
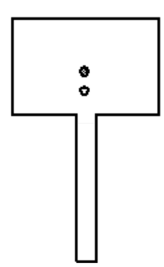
Center of mass *directly below* pivot



Extended free-body diagram



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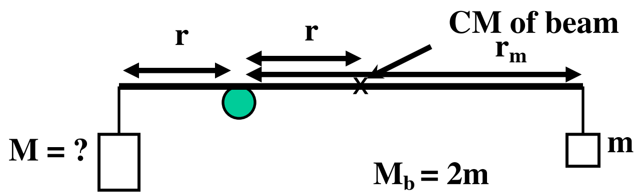
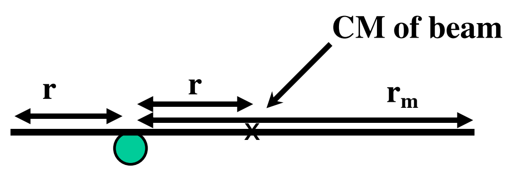
Example: A system composed of a beam and two masses rests at on a pivot point as shown in the diagram. The system is in mechanical equilibrium and $r_m = 3\ r$. a) what is the mass M in terms of m? b) what is the normal force of the pivot on the beam in terms of m?



extended free body diagram



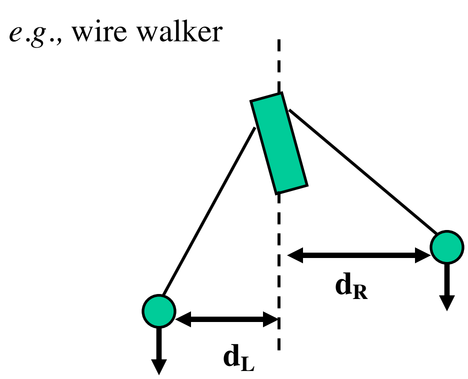
Vertical equilibrium? $\sum F =$

Rotational equilibrium? $\sum \tau =$

Examples of stable and unstable rotational equilibrium

- Wire walker – no net torque when figure vertical.
- Small deviations lead to a net ***restoring*** torque ➔ stable

Restoring torque

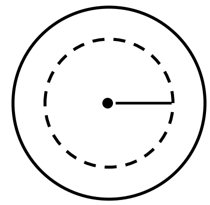


Consider displacing anticlockwise

- τ_R increases
- τ_L decreases

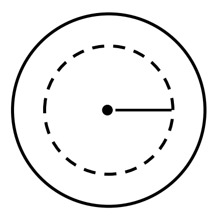
net torque causes clockwise rotation!

Recall:Rotations about fixed axis



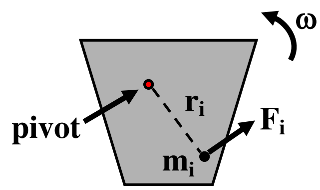
- Linear speed: $v = (2\pi r)/T = \omega r$.
  Quantity ω is called ***angular velocity***
- $\boldsymbol{\omega}$ is a vector! Use right hand rule to find direction of $\boldsymbol{\omega}$.
- Angular acceleration $\boldsymbol{\alpha} = \Delta\boldsymbol{\omega}/\Delta t$ is also a vector!
  - $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ *parallel* $\rightarrow$ angular speed increasing
  - $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ *antiparallel* $\rightarrow$ angular speed decreasing

Relating linear and angular kinematics



- Linear speed: $v = (2\pi r)/T = \omega r$
- Tangential acceleration: $a_{tan} = r\alpha$
- Radial acceleration: $a_{rad} = v^2/r = \omega^2 r$

Rotational Motion



* Particle *i*:
  $|v_i| = r_i\,\omega$ at 90° to r_i

* Newton's 2nd law:
  $m_i \Delta v_i / \Delta t = F_i^T$ ← component at 90° to r_i

* Substitute for v_i and multiply by r_i :
  $m_i r_i^2 \Delta\omega/\Delta t = F_i^T r_i = \tau_i$

* Finally, sum over all masses:

  $(\Delta\omega/\Delta t) \sum m_i r_i^2 = \sum \tau_i = \tau_{net}$

Discussion

$$(\Delta\omega/\Delta t) \sum m_i r_i^2 = \tau_{net}$$

α – angular acceleration

Moment of inertia, I

$$I\alpha = \tau_{net}$$

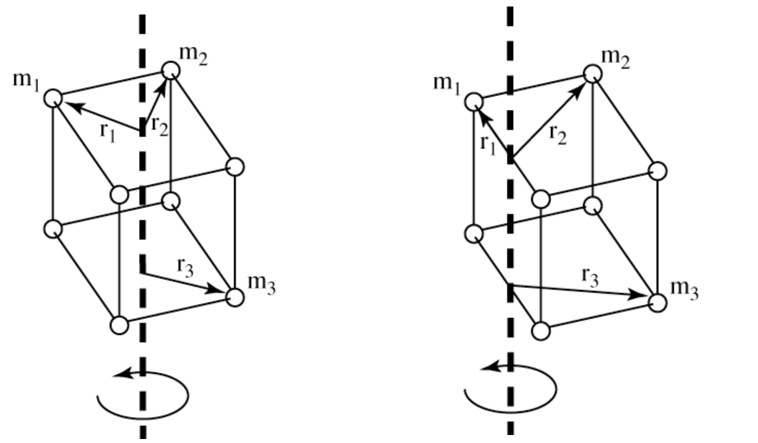
compare this with Newton's 2nd law

$$Ma = F$$

Moment of Inertia

$$\boxed{I = m_1 r_1^2 + m_2 r_2^2 + \ldots + m_N r_N^2 = \sum_{i=1}^{N} m_i r_i^2}$$

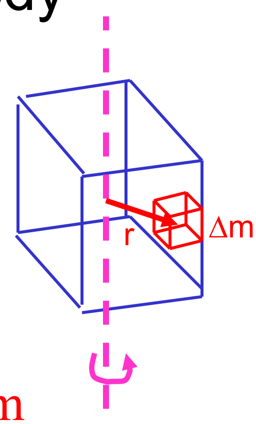
* I must be defined with respect to a particular axis



Moment of Inertia of Continuous Body

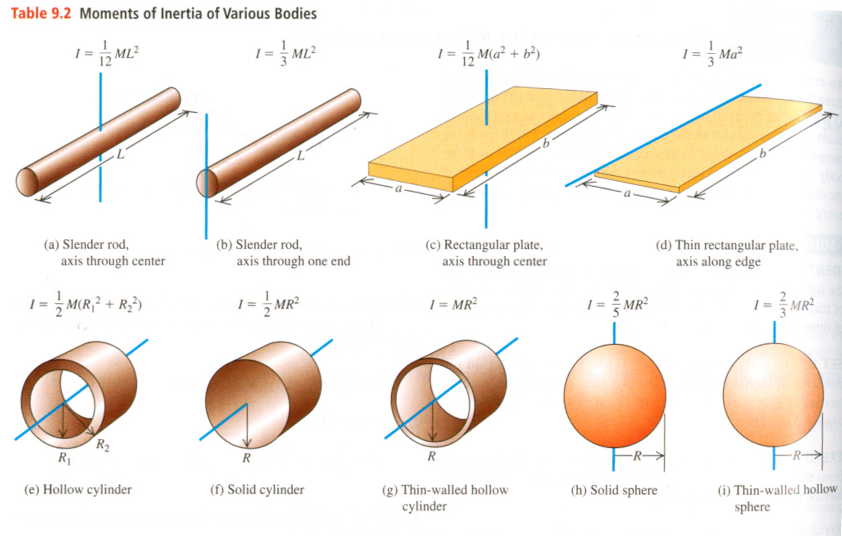
$\Delta m \mapsto 0$

$$\sum \Rightarrow \int$$



$$I = \sum_{i=1}^{N} m_i r_i^2 \quad \Rightarrow \quad I = \int r^2 dm$$

Tabulated Results for Moments of Inertia of some rigid, uniform objects



(from p.299 of *University Physics*, Young & Freedman)