Welcome back to Physics 215

Today's agenda:

- *Power*
- *Center of Mass*
- *Equilibrium of extended bodies*

Recall: Summary

- **Work** is defined as dot product of force with displacement vector
- Each individual force on an object gives rise to work done
- The **kinetic energy** only changes if **net work** is done on the object, which requires a **net force**

Power

Power = Rate at which work is done

$$\text{Average power} = \frac{W}{\Delta t}$$

$$\text{Inst. power} = \lim_{\Delta t \to 0} \frac{W}{\Delta t}$$

SG A sports car accelerates from zero to 30 mph in 1.5 s. How long does it take to accelerate from zero to 60 mph, assuming the power ($=\Delta W / \Delta t$) of the engine to be constant?

(Neglect losses due to friction and air drag.)

A. 2.25 s
B. 3.0 s
C. 4.5 s
D. 6.0 s

Power in terms of force and velocity

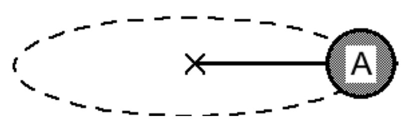
$$\text{Power} = \frac{W}{\Delta t} = \frac{\vec{F} \cdot \Delta \vec{s}}{\Delta t}$$

$$= \vec{F} \cdot \frac{\Delta \vec{s}}{\Delta t} = \vec{F} \cdot \vec{v}$$

Constant Power and kinetic energy:

$$\mathbf{P = \Delta W/\Delta t = \Delta K/\Delta t}$$

SG A ball is whirled around a horizontal circle at constant speed.



If air drag forces can be neglected, the power expended by the hand is:

A. positive
B. negative
C. zero
D. “Can’t tell.”

SG A locomotive accelerates a train from rest to a final speed of 40 mph by delivering constant power. If we assume that there are no losses due to air drag or friction, the acceleration of the train (while it is speeding up) is

A. decreasing
B. constant
C. increasing

Summary: Power

- Power:
  - $P = dW/dt$
  - $P = F\,v\cos\theta$
- It's the rate at which work or energy is transferred or transformed

Rigid Body rotation

OpenStax 10.1-10.3

Motion of Real Objects

- So far discussed motion of idealized point-like objects
- Saw that neglecting **internal** forces OK
  - only net **external** forces need to be considered for translational motion **of center of mass**
  - what about rotational motion?

Rigid Bodies

- Real extended objects can move in complicated ways (stretch, twist, etc.)
- Here, think of relative positions of each piece of body as fixed – idealize object as **rigid body**
- Can still undergo complicated motion (translational motion plus rotations)

Center of Mass

- Properties:
  - When a collection of particles making up an extended body is acted on by external forces, the ***center of mass*** moves as if all the mass of the body were concentrated there.
  - weight force can be considered to act vertically through ***center of mass***

Center of mass

for system of (point) objects:

$$\vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

$$= \frac{1}{M}\sum_i m_i\vec{r}_i$$

where $M = (m_1 + m_2 + \ldots)$

Points to note

- All real bodies are just collections of point-like objects (atoms)
- It is ***not*** necessary that CM lie within volume of body

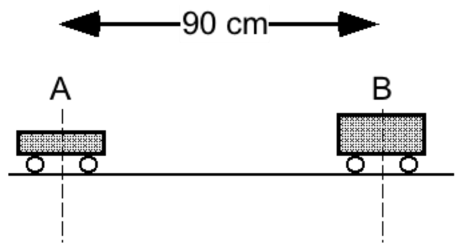
Center of mass board demos

- How can we find center of mass for funny-shaped object?
- Suspend from 2 points and draw in plumb lines.
- Where lines intersect yields CM

SG What is the center of mass of New York state?

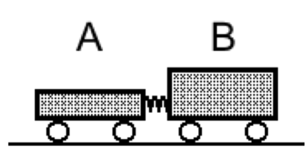
A. Binghamton
B. Syracuse
C. Utica
D. Albany

SG Two carts, A and B, of different mass ($m_B = 2\ m_A$) are placed a distance of 90 cm apart. The location of the center of mass of the two carts is



A. 30 cm to the right of cart A.
B. 45 cm to the right of cart A.
C. 60 cm to the right of cart A.
D. None of the above.

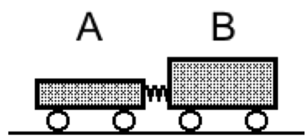
SG Two carts, A and B, with the same mass ($m_B = m_A$) are placed end-to-end on a low-friction track with a compressed spring between them. After the spring is released, cart A moves to the left; cart B, to the right.



Will the center of mass of the system

A. move to the right,
B. move to the left, or
C. stay at rest.
D. No clue.

SG Two carts, A and B, of different mass ($m_B = 3\ m_A$) are placed end-to-end on a low-friction track with a compressed spring between them. After the spring is released, cart A moves to the left; cart B, to the right.



Will the center of mass of the system

A. move to the right,
B. move to the left, or
C. stay at rest.
D. No clue.

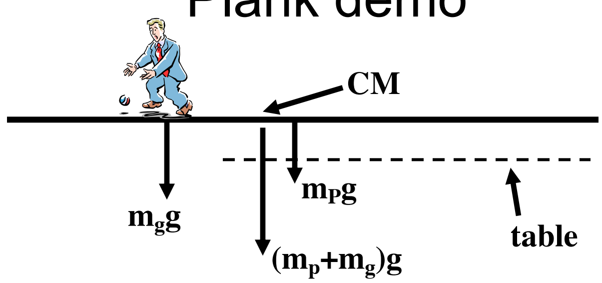
Use conservation of momentum

$$m_1\Delta v_1 + m_2\Delta v_2 = 0$$ -- *no external forces*

- Thus:
  a) $m_1\Delta r_1 + m_2\Delta r_2 = 0$
  b) $\Delta(m_1 r_1 + m_2 r_2) = 0$
  c) r_{CM} does not move!

Plank demo

Plank demo



- For equilibrium of plank, center of mass of person plus plank must lie above table
- Ensures normal force can act at same point and system can be thought of as point-like
- If CM lies away from table – equilibrium not possible – rotates!
- We will see later how to describe this demo using torques

SG A cart (of mass *m*) moving to the right at speed *v* collides with an ***identical*** stationary cart on a low-friction track. The two carts stick together after the collision and move to the right with speed 0.5 *v.*

Is the speed of the center of mass of the system after the collision

A. less than
B. equal to, or
C. greater than

the speed of the center of mass before the collision?

D. Not sure.

Situations with $F_{net} = 0$

- Consider 2 particles and $M = m_1+m_2$
- CM definition → $Mr_{CM} = m_1r_1 + m_2r_2$

$$M\Delta r_{CM}/\Delta t = m_1\Delta r_1/\Delta t + m_2\Delta r_2/\Delta t = m_1v_1 + m_2v_2$$

- RHS is total momentum

Thus, *velocity of center of mass is constant in absence of external forces!*

Conclusions

If there is ***no net force*** on a system,
the center of mass of the system:

- will stay at rest if it is initially at rest, or
- will continue to move with the same velocity if it is initially moving.

What about F_{ext} not zero?

Motion of center of mass of a system:

$$\sum \vec{F}_{\mathrm{ext}} = M\vec{a}_{\mathrm{CM}}$$

The center of mass of a system of point objects moves in the same way as a single object with the same total mass would move under the influence of the same net (external) force.

Throwing an extended object

- **Demo:** odd-shaped object – 1 point has simple motion = projectile motion
  - *center of mass*

- Total external force F_{ext}

$$a_{CM} = F_{ext} / M$$

- ***Translational*** motion of system looks like all mass is concentrated at CM

Equilibrium of extended object

- Clearly net force must be zero
- Also, if want object to behave as point at center of mass → **ALL forces acting on object must pass through CM**

Using the center of mass as the origin for a system of objects:

$$0 = \vec{r}_{\mathrm{CM}} = \frac{(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots)}{(m_1 + m_2 + \ldots)}$$

Therefore, the center of mass is the point which, if taken as the origin, makes:

$$(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots) = 0$$

Restatement of equilibrium conditions

$m_1r_1 + m_2r_2 = 0 \quad \rightarrow \quad m_1gr_1 + m_2gr_2 = 0$

- Thus, Σ force x displacement = 0
- quantity – force x displacement called ***torque (preliminary definition)***

Thus, equilibrium requires net torque to be zero

Conditions for equilibrium of an extended object

For an extended object that remains at rest and does not rotate:

- The net force on the object has to be zero.

$$\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}} = 0$$

- The net torque on the object has to be zero.

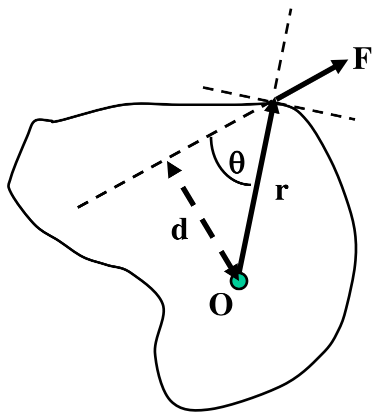
$$\vec{\tau}_{\text{net}} = \sum \vec{\tau} = 0$$

Preliminary definition of torque:

$$\tau = F\,d$$

The torque on an object with respect to a given pivot point and due to a given force is defined as the product of the force exerted on the object and the moment arm. The moment arm is the perpendicular distance from the pivot point to the line of action of the force.

Computing torque



$|\tau| = |F|d$
$= |F||r|\sin\theta$
$= (|F| \sin\theta)|r|$

component of force at 90^0 to position vector times distance

Definition of torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where ***r*** is the vector from the reference point (generally either the pivot point or the center of mass) to the point of application of the force ***F***.

$$|\vec{\tau}| = r\ F \sin\theta$$

where θ is the angle between the vectors ***r*** and ***F***.

Vector (or "cross") product of vectors

The vector product is a way to combine two vectors to obtain a *third vector* that has some similarities with multiplying numbers. It is indicated by a cross (×) between the two vectors.

The ***magnitude*** of the vector cross product is given by:

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

The ***direction*** of the vector **A**×**B** is *perpendicular* to the plane of vectors **A** and **B** and given by the right-hand rule.

Interpretation of torque

- Measures tendency of any force to cause rotation
- Torque is defined with respect to some origin – must talk about "torque of force about point X", etc.
- Torques can cause clockwise (-) or anticlockwise rotation (+) about pivot point