Welcome back to Physics 215

Today's agenda:

- *Impulse and momentum*
- *Energy*

Demo – swinging water bucket

- Does the water fall out?
- What is the FBD for the water?

Demo – swinging water bucket

- So why doesn't the water fall out?

- What if $mv^2/R < mg$ for the water?
- This is like a satellite in orbit around the earth
  - The inward force is _____________

Forces in circular motion summary:

- Draw a free body diagram
- Sum the forces as usual
  - There IS NOT AN "EXTRA" centripetal force
  - find F_{NET}(radial) and F_{NET}(other)
  - Velocity is NOT a force
- THEN figure out what F_{NET} (radial) has to be: in uniform circular motion
  - F_{NET}(radial) = ma
  - $a = v^2/r$

So far …

solved problems where forces don't change during the problem

what if they DO change?

Impulse

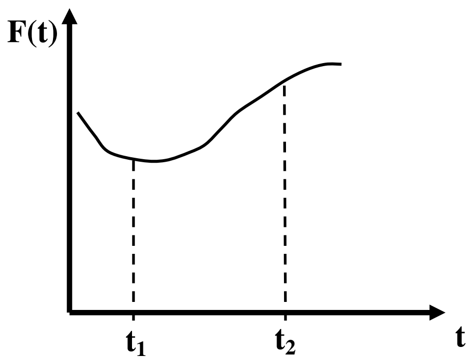
- Constant force F_{12} by object 1 on object 2 for a time Δt yields an **impulse**

$$I_{12} = F_{12}\,\Delta t$$

- In general, for a time varying force need to use this for small Δt and add:

$$I = \Sigma\, F(t)\,\Delta t =$$

Impulse for time varying forces



*** area under curve equals impulse**

Impulse → change in momentum

- Consider first constant forces ...
- Constant acceleration equation:

$$v_f = v_i + at$$

$$\downarrow$$

$$mv_f - mv_i = mat =$$

- If we call p = mv ***momentum*** we see that

$$\Delta p =$$

Definitions of *impulse* and *momentum*

Impulse imparted by object 1 on object 2:

$$I_{12} = F_{12}\Delta t$$

Momentum of an object:

$$\mathbf{p} = m\mathbf{v}$$

Impulse-momentum theorem

$$\mathbf{I_{net} = \Delta p}$$

The net impulse imparted to an object is equal to its change in its momentum.

SG Consider the **change in momentum** in these three cases:

A. A ball moving with speed *v* is brought to rest.

B. The same ball is projected from rest so that it moves with speed *v.*

C. The same ball moving with speed *v* is brought to rest and immediately projected backward with speed *v*.

In which case(s) does the ball undergo the largest magnitude of change in momentum?

A. Case A.
B. Case B.
C. Case C.
D. Cases A and B.

Demo: medicine ball and cart

- By Newton's 3rd law, the force on the ball is equal and opposite to the force on the student
- Acts for same time interval → equal and opposite changes in momentum

SG A student sitting on a cart is playing catch with an instructor standing on the ground. Assume everything is initially at rest, and the ball is thrown horizontally at the same velocity by both the student and instructor. Which action causes the student + cart system to move with the highest speed?

A. The student catching the ball from the instructor
B. The student throwing the ball to the instructor
C. Both actions result in the same speed for the student+cart system.

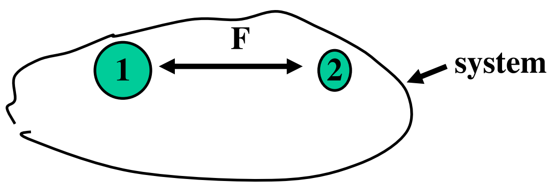
Newton's 3rd law and changes in momentum

If all external forces (weight, normal, etc.) cancel:

Conservation of momentum

- Assuming no net forces act on bodies there is no net impulse on composite system

- Therefore, no change in ***total*** momentum $\Delta(p_1 + p_2) = 0$



Conservation of momentum

(for a system consisting of two objects 1 and 2)

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

If the net (external) force on a system is zero, the total momentum of the system is constant.

Whenever two or more objects in an isolated system interact, the total momentum of the system remains constant

Conservation of momentum with carts

- *One cart with mass m_1 begins at rest $v_{1i} = 0$, and the other cart (with the same mass) has a velocity $v_{2i} = v$. After the two carts hit each other, what is the sum of the velocities of the two carts $v_{2f} + v_{1f}$?*

SG A student is sitting on a low-friction cart and is holding a medicine ball. The student then throws the ball at an angle of 60˚ (measured from the horizontal) with a speed of 10 m/s. The mass of the student (with the car) is 80 kg. The mass of the ball is 4 kg.

What is the final speed of the student (with car)?

1. 0 m/s
2. 0.25 m/s
3. 0.5 m/s
4. 1 m/s

Momentum is a vector!

$$\vec{p}_{A,\text{initial}} + \vec{p}_{B,\text{initial}} = \vec{p}_{A,\text{final}} + \vec{p}_{B,\text{final}}$$

• Must conserve components of momentum simultaneously

• In 2 dimensions:

Kinetic Energy

• Newton's Laws are *vector* equations
• Sometimes more appropriate to consider *scalar* quantities related to speed and mass

For an object of mass m moving with speed v:

$$K = (1/2)mv^2$$

• *Energy* of motion
• scalar!
• Measured in Joules -- J

Collisions

If two objects collide and the net force exerted on the system (consisting of the two objects) is zero, the sum of their momenta is constant.

$$\vec{p}_{A,\text{initial}} + \vec{p}_{B,\text{initial}} = \vec{p}_{A,\text{final}} + \vec{p}_{B,\text{final}}$$

The sum of their kinetic energies may or may not be constant.

Elastic and inelastic collisions

- If K is conserved – collision is said to be *elastic*

e.g., cue balls on a pool table

$$\mathrm{K}_{\mathrm{A,i}} + \mathrm{K}_{\mathrm{B,i}} \qquad \mathrm{K}_{\mathrm{A,f}} + \mathrm{K}_{\mathrm{B,f}}$$

- Otherwise termed *inelastic*

  e.g., lump of putty thrown against wall

$$\mathrm{K}_{\mathrm{A,i}} + \mathrm{K}_{\mathrm{B,i}} \qquad \mathrm{K}_{\mathrm{A,f}} + \mathrm{K}_{\mathrm{B,f}}$$

- Extreme case = *completely inelastic* -- objects stick together after collision

SG Cart A moving to the right at speed v collides with an identical stationary cart (cart B) on a low-friction track. The collision is *elastic* (*i.e.,* there is no loss of kinetic energy of the system).

What is each cart's velocity after colliding (considering velocities to the right as positive)?

	Cart A	Cart B
1	$-v$	$2v$
2	$-{}^{1}/_{3}\,v$	${}^{4}/_{3}\,v$
3	0	v
4	${}^{1}/_{3}\,v$	${}^{2}/_{3}\,v$

Check conservation of momentum and energy

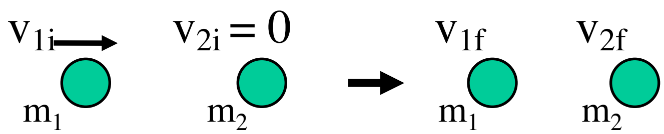
	Cart A *(m)*	Cart B *(m)*	Final momentum	Final kin. energy
1	$-v$	$2v$		
2	$-{}^{1}/_{3}\,v$	${}^{4}/_{3}\,v$		
3	0	v		
4	${}^{1}/_{3}\,v$	${}^{2}/_{3}\,v$		

Reading for next Tuesday

- Work and kinetic energy
- Reading 7.1-7.3

Sample Problem: At the intersection of Madison and University, a subcompact car with mass 950 kg traveling east on Madison collides with a pickup truck with mass 1900 kg that is traveling north on University and ran a red light. The two vehicles stick together as a result of the collision and, after the collision, the wreckage is sliding at 16.0 m/s in the direction 24° east of north. Calculate the speed of each vehicle before the collision. You can ignore friction forces between the vehicles and the wet road.

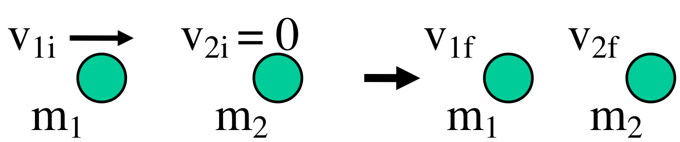
Elastic collision of two masses



Momentum → $m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$

Energy → $(1/2)m_1 v_{1i}^2 + 0 = (1/2)m_1 v_{1f}^2 + (1/2)m_2 v_{2f}^2$

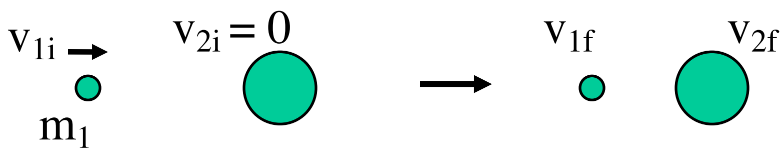
Special cases: (i) $m_1 = m_2$



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Special cases: (ii) $m_1 << m_2$



$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Gravitational Potential Energy

For an object of mass m near the surface of the earth:

$$U_g = mgh$$

- h is height above arbitrary reference line
- Measured in Joules -- J (like kinetic energy)

Total energy for object moving under gravity

$E = U_g + K =$ constant

* E is called the (mechanical) energy

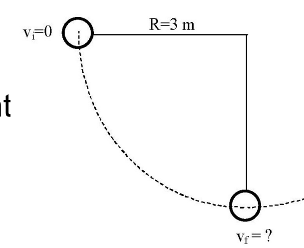
* It is conserved:

$$(\tfrac{1}{2})\, mv^2 + mgh = \text{constant}$$

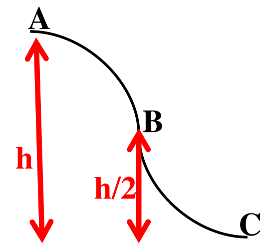
OR

Sample problem: A ball of mass m=7 kg attached to a massless string of length R=3 m is released from the position shown in the figure below. (a) Find magnitude of velocity of the ball at the lowest point on its path.

Small group:
(b) Find the tension in the string at that point

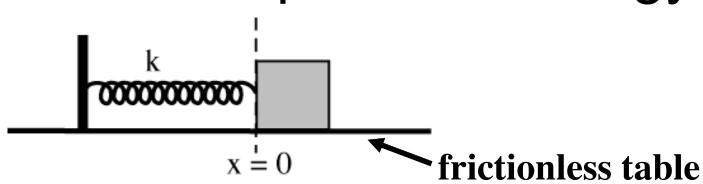


SG If the velocity at B is v, then what is the velocity at C?



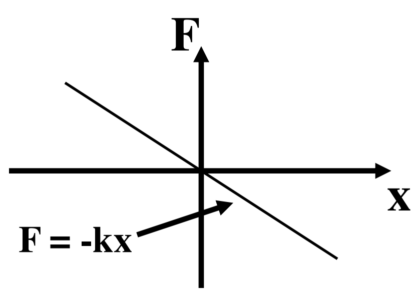
A. 2v
B. v
C. sqrt(2) v
D. v/sqrt(2)
E. None of the above

Springs -- Elastic potential energy



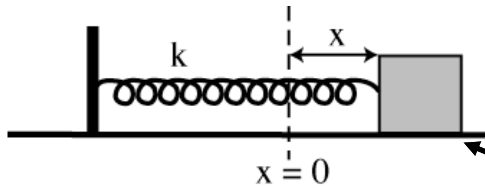
Force F = -kx (Hooke's law)

Area of triangle lying under straight line graph of F *vs*. x = (1/2)(+/-x)(-/+kx)



$\mathbf{U_s = (1/2)kx^2}$

(Horizontal) Spring



frictionless table

- x = displacement from relaxed state of spring

- Elastic potential energy stored in spring: $\mathbf{U_s = (1/2)kx^2}$

$$\mathbf{(1/2)kx^2 + (1/2)mv^2 = constant}$$

SG A 0.5 kg mass is attached to a spring on a horizontal frictionless table. The mass is pulled to stretch the spring 5.0 cm and is released from rest. When the mass crosses the point at which the spring is not stretched, x = 0, its speed is 20 cm/s. If the experiment is repeated with a 10.0 cm initial stretch, what speed will the mass have when it crosses x = 0 ?

1. 40 cm/s
2. 0 cm/s
3. 20 cm/s
4. 10 cm/s

Work, Energy

- Newton's Laws are ***vector*** equations

- Sometimes easier to relate speed of a particle to how far it moves under a force – a single equation can be used – need to introduce concept of ***work***

What is work?

- Assume ***constant*** force in 1D

- Consider:

$$v_F^2 = v_I^2 + 2a\ \Delta x$$

- Multiply by m/2 →

$$(1/2)mv_F^2 - (1/2)mv_I^2 = ma\Delta x$$

- But: F = ma

$$\rightarrow (1/2)mv_F^2 - (1/2)mv_I^2 = F\Delta x$$

Work-Kinetic Energy theorem (1)

$$(1/2)mv_F^2 - (1/2)mv_I^2 = Fs$$

Points: $s = \Delta x$ = displacement (for 2D)

- W = Fs → defines **work done** on particle

  = force times displacement

- K = (1/2)mv^2 → defines **kinetic energy**

  =1/2 mass times square of v