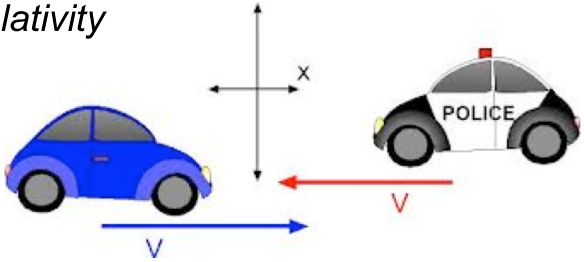
Welcome back to Physics 215

Today's agenda:

- *Relative Motion*
- *Special relativity*
- *Forces*



Midterm 1: next Tuesday (9/24)

- In Phys 208 (here!) at the usual lecture time
- Material covered:
  - **Textbook** chapters 1 – 4
  - **Lectures** up through this past Tuesday (slides online)
  - **Wed/Fri recitation activities**
  - **Homework assignments**
- You will be given a **formula sheet** at the exam. A copy of this sheet will be available on the course website
- You should bring a **calculator**, but you must bring your own, and it can not be a phone. You may not store any equations in memory, and midterm proctors may request to see your calculator during the exam.
- Pizza and Study session: Tonight!!! 6-8pm in Physics 202/204

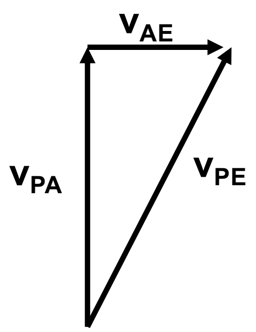
Recall from last time

- If we want to use (inertial) moving frames of reference, then velocities are **not** the same in different frames
- However **constant velocity** motions are always seen as **constant velocity**
- There is a simple way to relate velocities measured by different frames.

Relative Motion in 2D

- Consider airplane flying in a crosswind
  - velocity of plane relative to air, $\mathbf{v}_{PA}$ = 240 km/h N
  - wind velocity, air relative to earth, $\mathbf{v}_{AE}$ = 100 km/h E
  - what is velocity of plane relative to earth, $\mathbf{v}_{PE}$?

$$\mathbf{v}_{PE} = \mathbf{v}_{PA} + \mathbf{v}_{AE}$$



Acceleration is same for all inertial FOR!

- We have:

$$v_{PA} = v_{PB} + v_{BA}$$

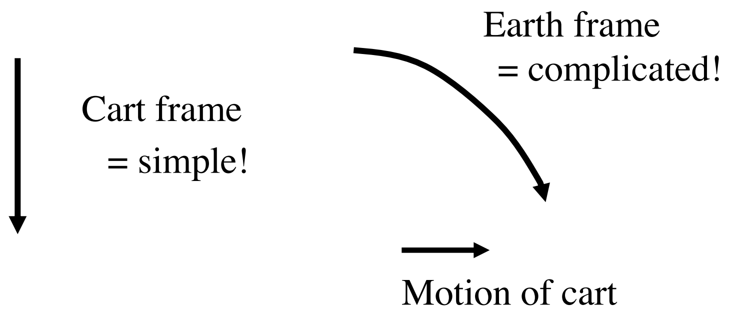
- For velocity of P measured in frame A in terms of velocity measured in B

→$\Delta v_{PA}/\Delta t = \Delta v_{PB}/\Delta t$ since v_{BA} is *constant*

→Thus acceleration measured in frame A or frame B is same!

Relative Motion in 2D

- Motion may look quite different in different inertial frames, e.g., ejecting ball from moving cart



Earth frame
= complicated!

Cart frame
= simple!

Motion of cart

SG : While standing still, Otto shoots a basketball into the air. The initial x-component of the basketball velocity is 2m/s, while the y-component of the velocity is 1.5 m/s. Just as he is shooting, a cameraman rolls by in a car at moving at 2 m/s (along the x-axis). What is the apparent launch angle in the frame of the camera?

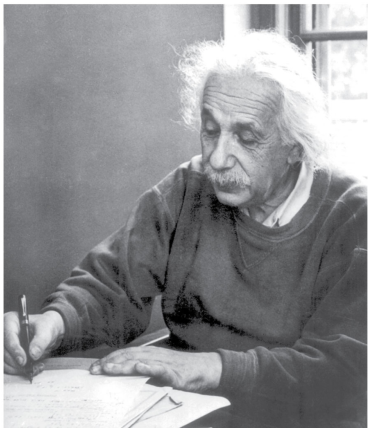
A. Less than 90 degrees
B. More than 90 degrees
C. Exactly 90 degrees
D. We don’t have enough information to answer this question

Space

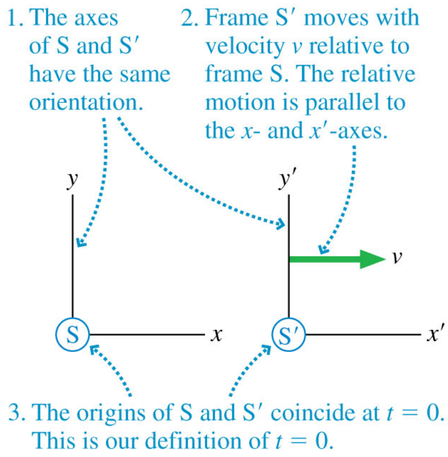
The physical length of an object is *less* when the object is moving in a reference frame than when it is at rest in that reference frame. This is **length contraction.**

To us, the Fermilab Accelerator is 3.9 miles in circumference. To protons in the accelerator, moving at 0.999999*c*, the circumference is only 30 feet.

- Albert Einstein (1879–1955) was one of the most influential thinkers in history.
- Einstein’s first paper on *Special Relativity*, in 1905, dealt exclusively with inertial reference frames.
- Ten years later, Einstein published a more encompassing theory of *General Relativity* that considered accelerated motion and its connection to gravity.



- We are covering only the theory of Special Relativity.



1. The axes of S and S′ have the same orientation.

2. Frame S′ moves with velocity v relative to frame S. The relative motion is parallel to the x- and x'-axes.

3. The origins of S and S′ coincide at $t = 0$. This is our definition of $t = 0$.

- The figure shows two reference frames called S and S′.
- The coordinate axes in S are x, y, z and those in S′ are x', y', z'.
- Frame S′moves with velocity v relative to S.

- According to Maxwell's theory of electromagnetism, light waves travel with speed

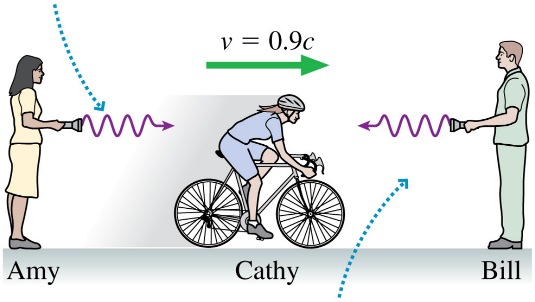
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

- Maxwell's equations are true in all inertial reference frames.
- Therefore, light travels at speed *c* in all inertial reference frames.

Principle of relativity All the laws of physics are the same in all inertial reference frames.

- Because of the principle of relativity, light travels at speed c in *all* inertial reference frames.
- Amy, Cathy, and Bill all measure both light waves to be moving at speed *c* in their own reference frame!

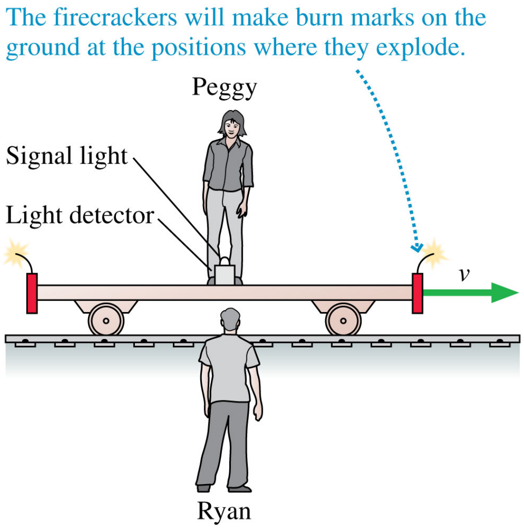
This light wave leaves Amy at speed c relative to Amy. It approaches Cathy at speed c relative to Cathy.



This light wave leaves Bill at speed c relative to Bill. It approaches Cathy at speed c relative to Cathy.

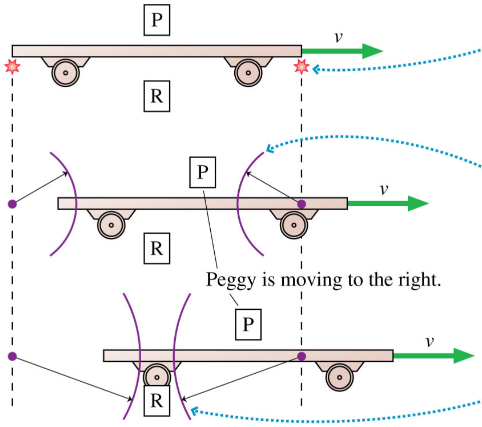
SG: A firecracker explodes high overhead. You notice a slight delay between seeing the flash and hearing the boom. At what time does the event "firecracker explodes" occur?

A. At the instant you hear the boom.
B. At the instant you see the flash.
C. Very slightly before you see the flash.
D. Very slightly after you see the flash.
E. There's no unique answer because it depends on the observer.



- The figure shows a railroad car traveling to the right with a velocity v.
- A firecracker is tied to each end of the car, just above the ground.
- The two events are *simultaneous* in Ryan's reference frame.

Below is the sequence of events in Ryan's reference frame.



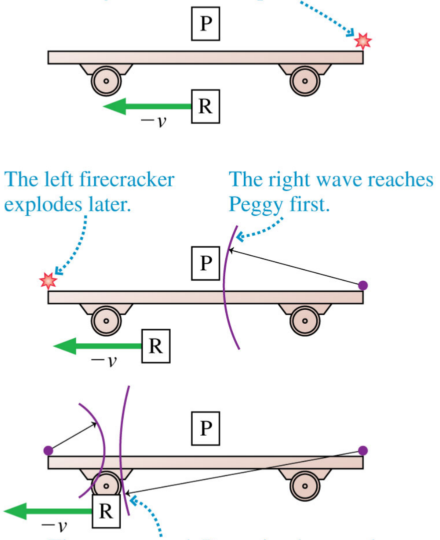
Explosions are simultaneous. Burn marks are equal distances from Ryan.

The waves are spheres centered on the burn marks because the light speed of both is c.

Peggy is moving to the right.

The waves reach Ryan simultaneously. The right wave has already passed Peggy and been detected. The left wave has not arrived.

The right firecracker explodes first.



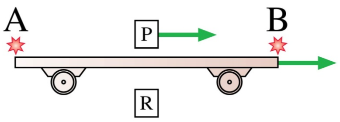
The left firecracker explodes later.

The right wave reaches Peggy first.

The waves reach Ryan simultaneously. The left wave has not reached Peggy.

- The figure shows the sequence of events in Peggy's reference frame.
- The two events are *not* simultaneous in Peggy's reference frame!
- This is called the relativity of simultaneity.

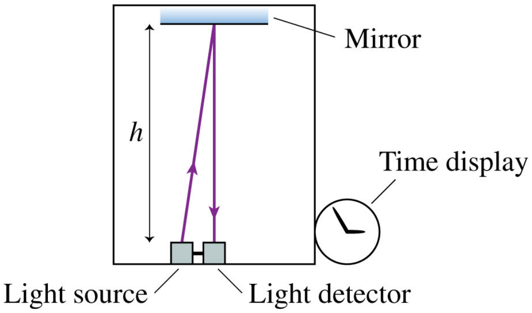
SG: Peggy is standing at the center of a railroad car as it passes Ryan. Firecrackers A and B at the ends of the car explode. A short time later, flashes from the two explosions reach Peggy at the same instant. In Ryan's reference firecracker A explodes ___ firecracker B.



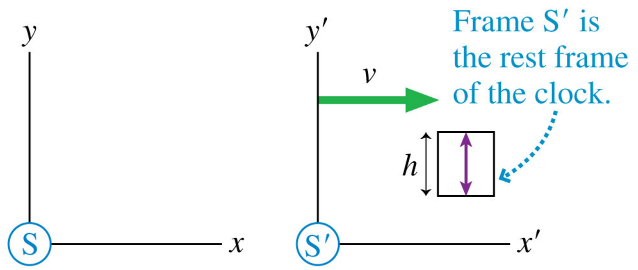
A. before

B. at the same time as

C. after

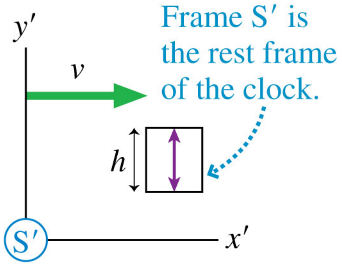


- The figure shows a special clock called a *light clock*.
- The light clock is a box with a light source at the bottom and a mirror at the top, separated by distance h.
- The light source emits a very short pulse of light that travels to the mirror and reflects back to a light detector beside the source.
- The clock advances one "tick" each time the detector receives a light pulse.



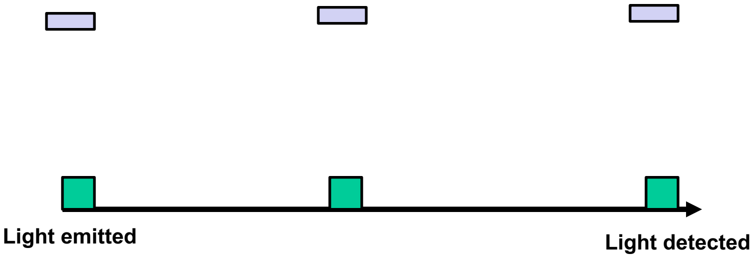
Frame S′ is the rest frame of the clock.

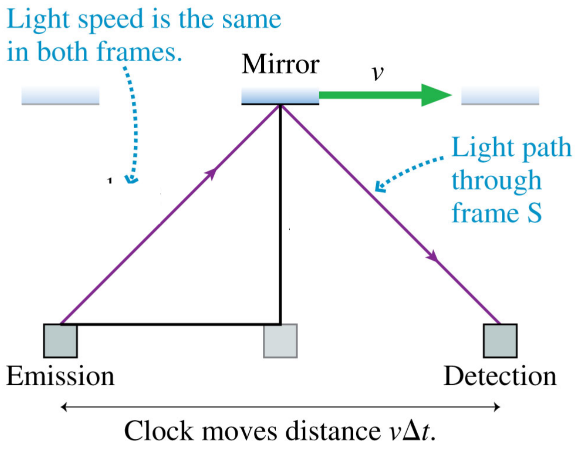
- Consider a light clock at rest in reference frame S'.
- We call this the rest frame of the clock.
- Reference frame S' moves to the right with velocity v relative to reference frame S.



Frame S′ is the rest frame of the clock.

SG: Draw a picture of the path of the light in the frame of reference of someone on the ground. Label distances in terms:
c, v , h and Δt





- The figure shows the light clock as seen in frame S.

- In frame S, light takes time Δt to travel along the hypotenuse from one mirror to the other.
- We can use the Pythagorean theorem to write:
- Solving for Δt gives:
- If we define the dimensionless number $\beta = v/c$:

$$\Delta t = \frac{\Delta t'}{}$$

- The time interval between two events that occur at the *same position* is called the proper time $\Delta\tau$.
- The proper time is measured with a single clock that is present at both events.
- An inertial reference frame moving with velocity $v = \beta c$ relative to the proper-time frame must use two clocks to measure the time interval: one at the position of the first event, the other at the position of the second event.
- The time interval between the two events in this frame is:

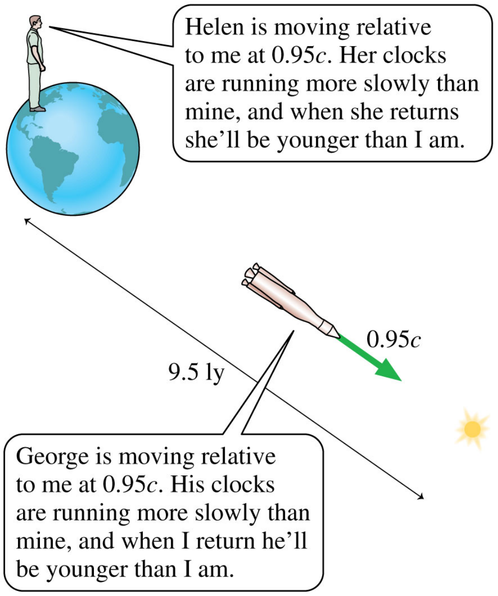
$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}} \geq \Delta\tau \qquad \text{(time dilation)}$$

- George and Helen are twins.
- On their 25th birthday, Helen departs on a starship voyage to a distant star.
- Helen's starship travels at speed $0.95c$ to a star that is 9.5 light years from earth.
- Upon arriving, she immediately turns around and heads home at $0.95c$.
- According to George, it takes Helen 20 years to travel out and back.
- George will be $25 + 20 = 45$ years old when his sister Helen returns.

- In George’s reference frame, Helen’s moving clock will run *more slowly* than his stationary clock.
- Helen’s clock travels with her, so it measures the proper time $\Delta\tau$:

$$\Delta t_{\mathrm{H}} =$$

- Helen will be 25 + 6.25 = just over 31 years old when she returns, about 14 years *younger* than her twin brother!



SG question:

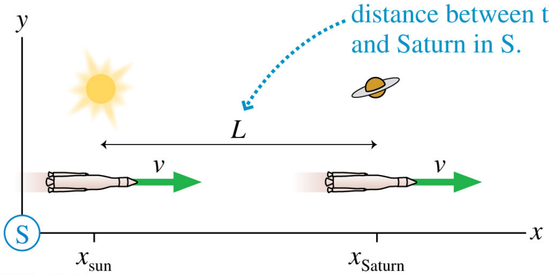
- In Helen’s reference frame, it is *George’s* clock that is moving, so it runs more slowly than hers.
- According to Helen’s calculations, George will be *younger* than her when she returns!
- Who is correct?

- Note that Helen’s reference frame is *not* inertial.
- Helen must accelerate at the beginning, middle, and end of her journey.
- The principle of relativity applies only to inertial reference frames.
- Our equations for time dilation are only valid for inertial reference frames.
- Thus George’s analysis and calculations are correct.
- Helen’s analysis and calculations are *not* correct because she was trying to apply an inertial reference frame result while traveling in a noninertial reference frame.
- Helen is *younger* than George when she returns.

What about lengthscales?

How far is the sun from Saturn?

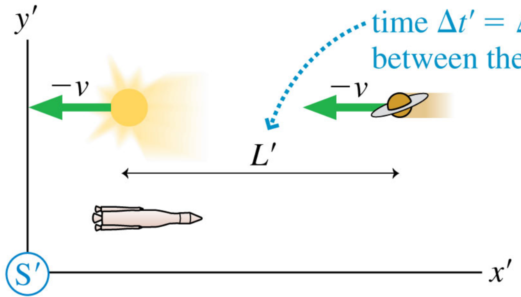
- Consider a rocket which travels from the sun to Saturn at speed v.
- In the reference frame S of the solar system, the distance traveled is L, and $v = L/\Delta t$, where Δt is the time for the rocket to make the journey in the S frame.



The rocket moves distance L in time Δt. This is the distance between the sun and Saturn in S.

- In the reference frame S', the rocket is at rest and the solar system moves to the left at speed v.
- The relative speed between S and S' is the *same* for both reference frames:

$$v = \frac{L}{\Delta t} = \frac{L'}{\Delta t'}$$

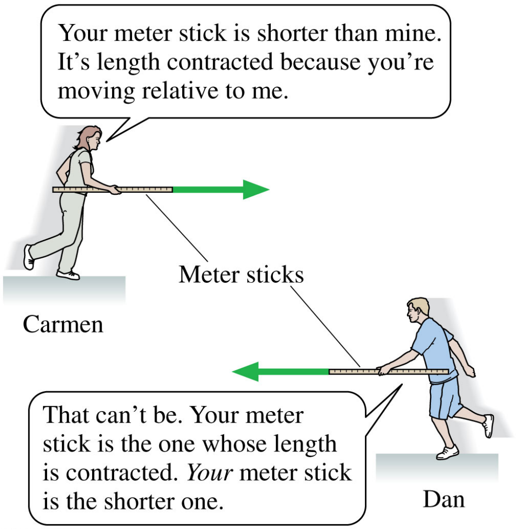


Saturn moves distance L' in time $\Delta t' = \Delta\tau$. This is the distance between the sun and Saturn in S'.

- The time interval $\Delta t'$ measured in frame S' is the proper time $\Delta\tau$ because both events occur at the same position in frame S' and can be measured by one clock.
- Since the speed is the same in both frames:

$$\frac{L}{\Delta t} = \frac{L'}{\Delta\tau} =$$

- The Δt cancels, and the distance L' in frame S' is:
- where ℓ is the proper length measured in the frame where the objects are at rest.
- The length of any object is *less* when it is measured in a reference frame in which the object is moving.



- Carmen and Dan each carry meter sticks, and run past each other, in opposite directions, at a relative speed $v = 0.9c$.
- Dan's meter stick can't be both longer and shorter than Carmen's meter stick.
- Is this another paradox?

- Relativity allows us to compare the *same* events as they're measured in two different reference frames.
- But the events by which Dan measures the length of Carmen's meter stick are *not the same events* as those by which Carmen measures the length of Dan's meter stick.
- In Dan's reference frame, Carmen's meter stick has been length contracted, and is less than $1\ \mathrm{m}$ in length.
- In Carmen's reference frame, Dan's meter stick has been length contracted, and is less than $1\ \mathrm{m}$ in length.
- There's no conflict between their measurements!

Recall: acceleration is same for all inertial FOR!

- We have:

$$v_{PA} = v_{PB} + v_{BA}$$

- For velocity of P measured in frame A in terms of velocity measured in B

→$\Delta v_{PA}/\Delta t = \Delta v_{PB}/\Delta t$ since v_{BA} is *constant*

→Thus acceleration measured in frame A or frame B is same!