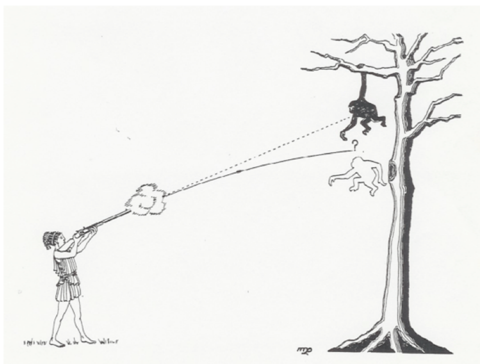
Welcome back to Physics 215!



Today's agenda:

- *Velocity and acceleration in two-dimensional motion*
- *Motion under gravity -- projectile motion*
- *Acceleration on curved path*

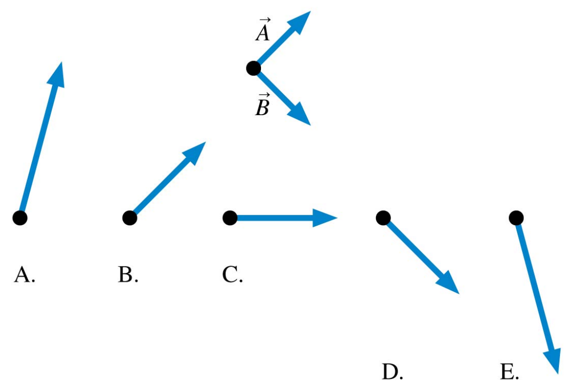
Current assignments

Due Fri

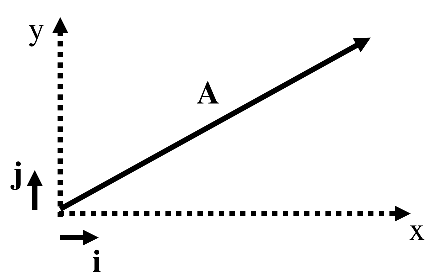
- *HW#3*
- *SAGE assignment*

SG 2-2.2:

Which of the vectors in the second row shows $2\vec{A} - \vec{B}$?



Components



$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y}$

$\mathbf{A} = a_x\mathbf{i} + a_y\mathbf{j}$

i = *unit vector* in x direction

j = *unit vector* in y direction

<u>Projection</u> of **A** along coordinate axes

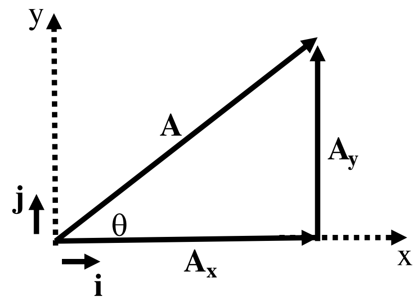
a_x, a_y = *components* of vector **A**

More on vector components

- Relate components to direction (2D):

  $a_x = |\mathbf{A}|\cos\theta$,

  $a_y = |\mathbf{A}|\sin\theta$



or

- *Direction:* $\tan\theta = a_y/a_x$
- *Magnitude:* $|A|^2 = a_x^2 + a_y^2$

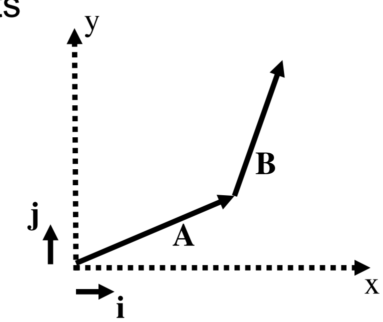
SG question: A bird is flying along a straight line in a direction somewhere East of North. After the bird has flown a distance of 2.5 miles, it is 2 miles North of where it started.

How far to the East is it from its starting point?

A. 0 miles
B. 0.5 miles
C. 1.0 mile
D. 1.5 miles

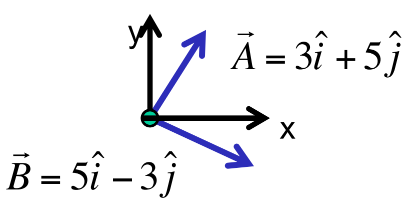
Why are components useful?

- *Addition:* just add components
  e.g. if **C** = **A** + **B**
  $c_x = a_x + b_x$; $c_y = a_y + b_y$
- *Subtraction* similar
- *Multiplying* a vector by a number – just multiply components: if **D** = n*__A__
  $d_x = n*a_x$; $d_y = n*a_y$
- Even more useful in 3 (or higher) dimensions



SG:

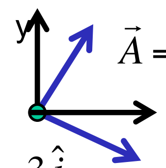
What is the sum of $\vec{A} + \vec{B}$?



A. 8 **i** + 8 **j**
B. 15 **i** – 15 **j**
C. 8 **i** - 2 **j**
D. 8 **i** + 2 **j**

SG:

What is the length of $\vec{A} + \vec{B}$?



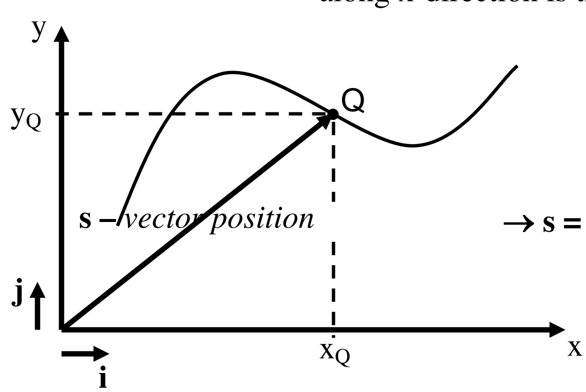
$\vec{A} = 3\hat{i} + 5\hat{j}$

$\vec{B} = 5\hat{i} - 3\hat{j}$

A. 6
B. 10
C. sqrt(64)
D. sqrt(68)

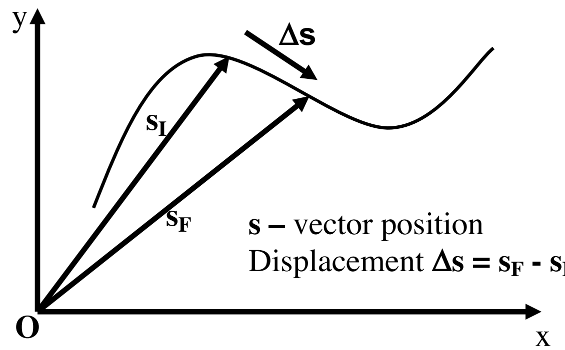
2D Motion in components

Note: component of position vector along x-direction is the x-coordinate!



→ $\mathbf{s} = x_Q\mathbf{i} + y_Q\mathbf{j}$

Displacement in 2D Motion



s – vector position
Displacement $\Delta \mathbf{s} = \mathbf{s}_F - \mathbf{s}_I$, also a vector!

Describing motion with vectors

- Positions and displacements

  $s, \quad \Delta s = s_F - s_I$

- Velocities and changes in velocity:

  $v_{av} = \underline{\qquad}, \quad v_{inst} = \lim_{\Delta t \to 0} \underline{\qquad}$

  $\Delta v = v_F - v_I$

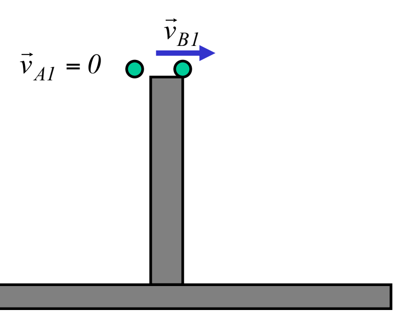
- Acceleration: $a_{av} = \underline{\qquad}, \quad a_{inst} = \lim_{\Delta t \to 0} \underline{\qquad}$

2D Motion in components

- x and y motions *decouple*
- $v_x = \Delta x/\Delta t$; $v_y = \Delta y/\Delta t$
- $a_x = \Delta v_x/\Delta t$; $a_y = \Delta v_y/\Delta t$
- If acceleration is only non-zero in 1 direction, can choose coordinates so that 1 component of acceleration is zero
  – e.g., motion under gravity

SG Question:

Ball A is released from rest. Another identical ball, ball B is thrown horizontally at the same time and from the same height. Which ball will reach the ground first?



A. Ball A
B. Ball B
C. Both balls reach the ground at the same time.
D. The answer depends on the initial speed of ball B.

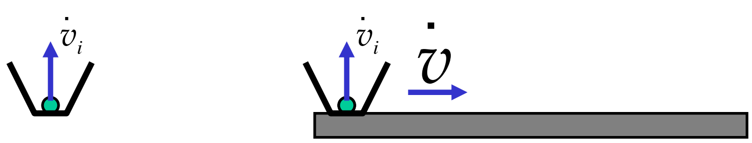
SG

A 100 g ball rolls off a table and lands 2.0 m from the base of the table. A 200 g ball rolls off the same table with the same speed. It lands at distance

1. 1.0 m.
2. Between 1 m and 2 m.
3. 2.0 m.
4. Between 2 m and 4 m.
5. 4.0 m.

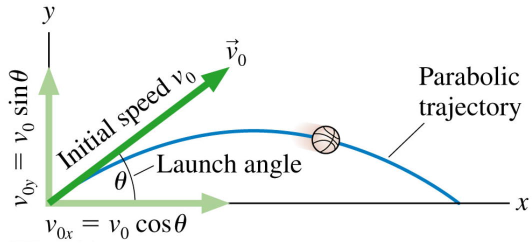
SG A ball is ejected vertically upward from a cart at rest. The ball goes up, reaches its highest point and returns to the cart.

In a second experiment, the cart is moving at constant velocity and the ball is ejected in the same way, where will the ball land?



A. In front of the cart.
B. Behind the cart.
C. Inside the cart.
D. The outcome depends on the speed of the cart.

- The start of a projectile's motion is called the *launch*.
- The angle θ of the initial velocity v_0 above the x-axis is called the **launch angle**.



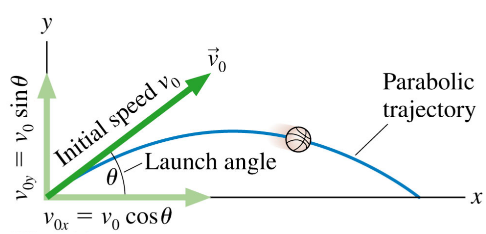
- The initial velocity vector can be broken into components.

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

where v_0 is the initial speed.

- Gravity acts downward.
- Therefore, a projectile has no horizontal acceleration.
- Thus:



$$a_x = 0$$
$$a_y = -g$$
(projectile motion)

- The vertical component of acceleration a_y is $-g$ of free fall.
- The horizontal component of a_x is zero.
- Projectiles are in free fall.

Motion under gravity

y- motion

$a_y = -g$

$v_y = v_{0y} - gt$

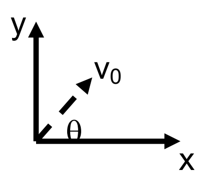
$y = y_0 + v_{0y}t - (1/2)gt^2$

x- motion

$a_x = 0$

$v_x = v_{0x}$

$x = x_0 + v_{0x}t$



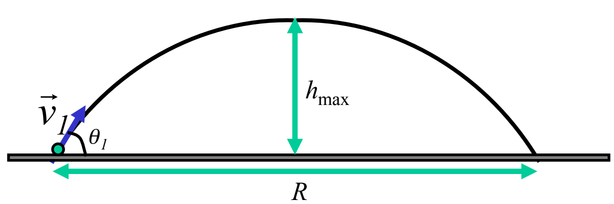
$v_{0y} = v_0 \sin(\theta)$

$v_{0x} = v_0 \cos(\theta)$

Projectile motion...

Example problem

A ball is thrown at 40° to vertical with a speed of 7 m/s. Assuming g=10 m/s^2, how far away does the ball land?



Example problem

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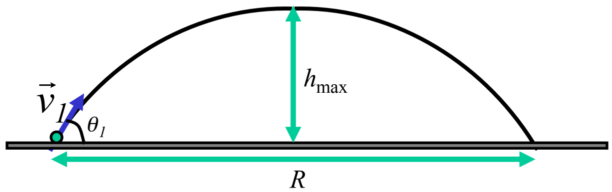


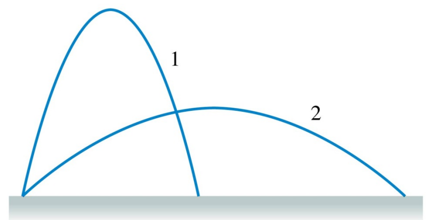
Maximum height and range (only works on FLAT ground)

$$2\cos\theta\sin\theta = \sin(2\theta) \qquad R = \frac{v_1^2 \sin(2\theta_1)}{g}$$

Challenge: Derive the formula for the maximum height: $h_{max} = \frac{v_0^2 \sin^2\theta}{2g}$

SG

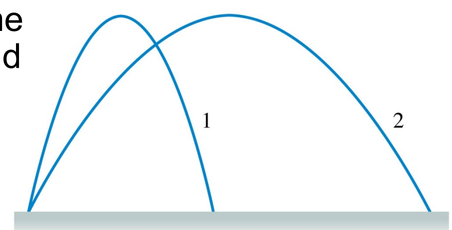
Projectiles 1 and 2 are launched over level ground with the same speed but at different angles. Which hits the ground first? Ignore air resistance.



1. Projectile 1 hits first.
2. Projectile 2 hits first.
3. They hit at the same time.
4. There's not enough information to tell.

SG

Projectiles 1 and 2 are launched over level ground with different speeds. Both reach the same height. Which hits the ground first? Ignore air resistance.

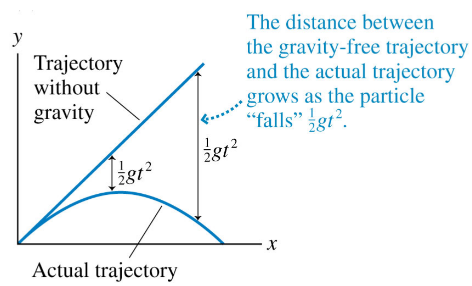


1. Projectile 1 hits first.
2. Projectile 2 hits first.
3. They hit at the same time.
4. There's not enough information to tell.

Hunter and "coconut" demo

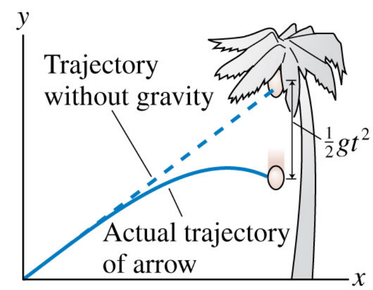
Why does the arrow hit the coconut?

- Without gravity, the arrow would follow a straight line.
- Because of gravity, the arrow at time *t* has "fallen" a distance $\frac{1}{2}gt^2$ below this line.
- The separation grows as $\frac{1}{2}gt^2$, giving the trajectory its parabolic shape.

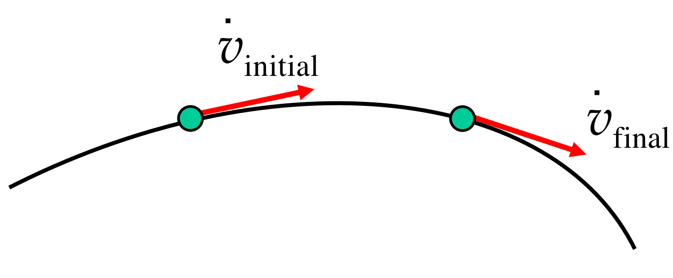


Why does the arrow hit the coconut?

- Had the coconut stayed on the tree, the arrow would have curved under its target as gravity causes it to fall a distance $\frac{1}{2}gt^2$ below the straight line.
- But $\frac{1}{2}gt^2$ is also the distance the coconut falls while the arrow is in flight.
- So *yes*, the arrow hits the coconut!

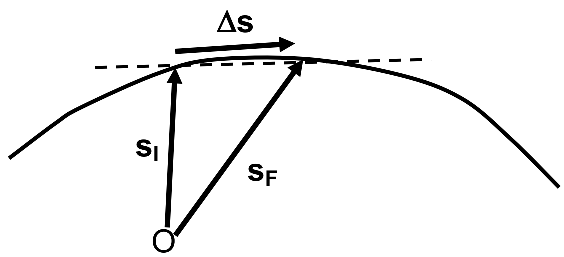


Motion on a curved path at constant speed



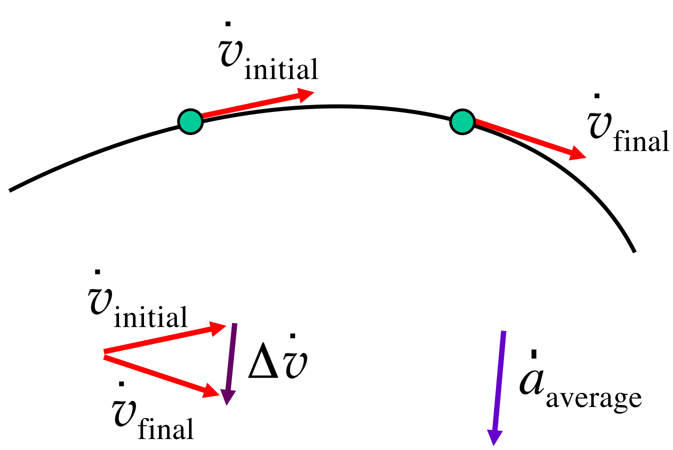
Is the acceleration of the object equal to zero?

Velocity is tangent to path



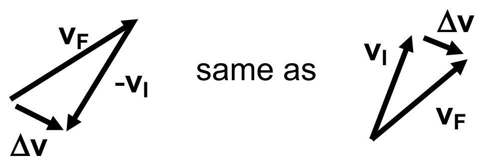
$\mathbf{v} = \Delta\mathbf{s}/\Delta t$ lies along dotted line. As $\Delta t \rightarrow 0$ direction of **v** is **tangent** to path

Motion on a curved path at constant speed



Subtracting vectors

Recall that $\mathbf{v_F} + (-\mathbf{v_I}) = \Delta\mathbf{v}$

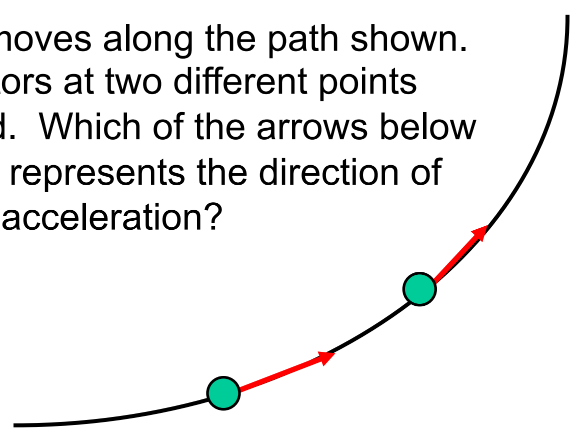


For an object moving at constant *speed* along a curved path, the acceleration is ***not*** zero.

3-1.7 For which of the following motions of a car does the net change in velocity vector have the greatest magnitude? (All motions occur at the same constant speed.)

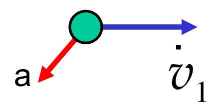
1. A 90˚ right turn at constant speed
2. A U-turn at constant speed
3. A 270˚ turn on a highway on-ramp
4. The change in velocity is zero for all three motions.

3-1.8 A car moves along the path shown. Velocity vectors at two different points are sketched. Which of the arrows below most closely represents the direction of the average acceleration?



3-1. 9 A child is riding a bicycle on a level street. The velocity and acceleration vectors of the child at a given time are shown.

Which of the following velocity vectors may represent the velocity at a later time?



1. 
2. 
3. 
4. 

Summary

- For motion at constant speed, instantaneous acceleration vector is perpendicular to velocity vector
- Points ``inward''
- What is the magnitude of the acceleration vector?

Reading assignment

- Knight Ch. 4.5-4.7 : Circular motion
- Prelecture 3-2