Boulder Condensed Matter Summer School
Theoretical Biophysics

Active Living Matter + Tissue Mechanics
July 15-19, 2019

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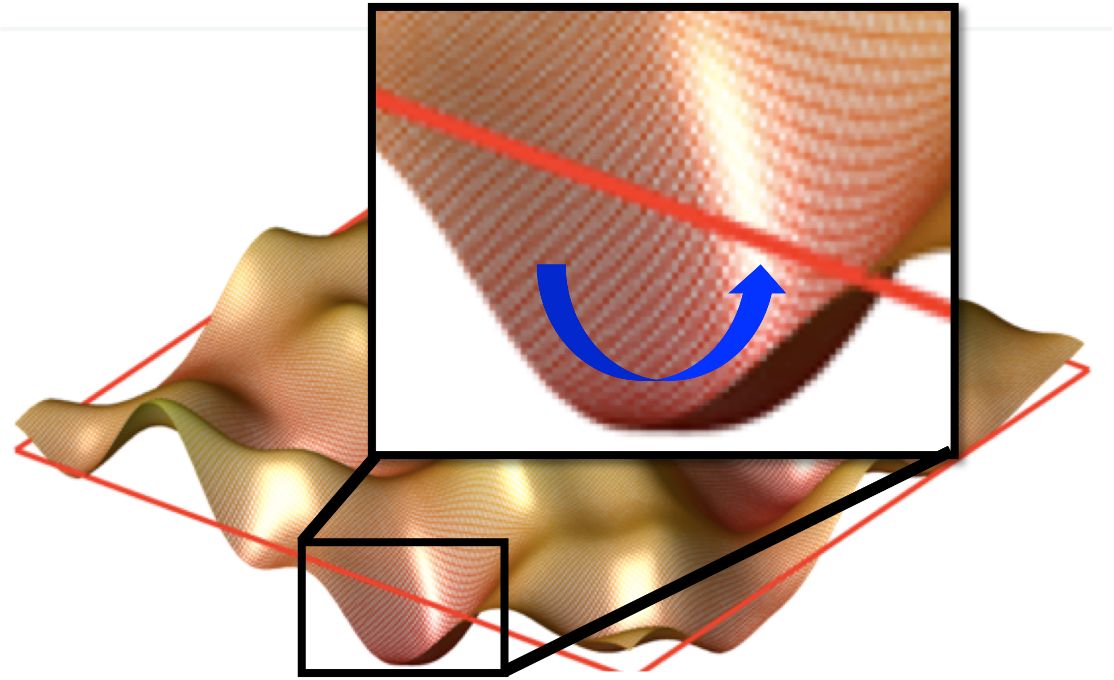
lecture notes:

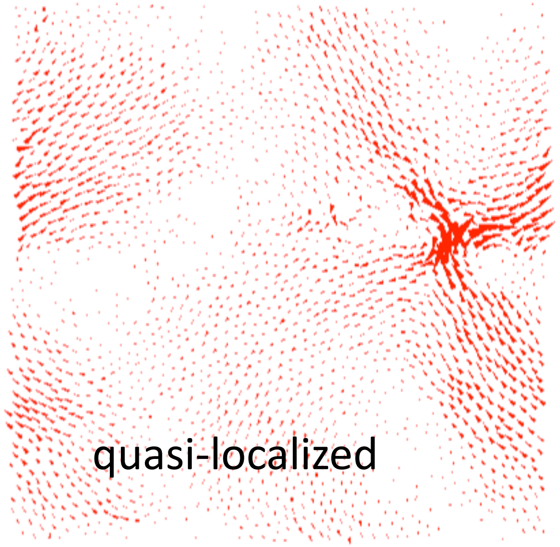
What are the linear low-energy excitations?

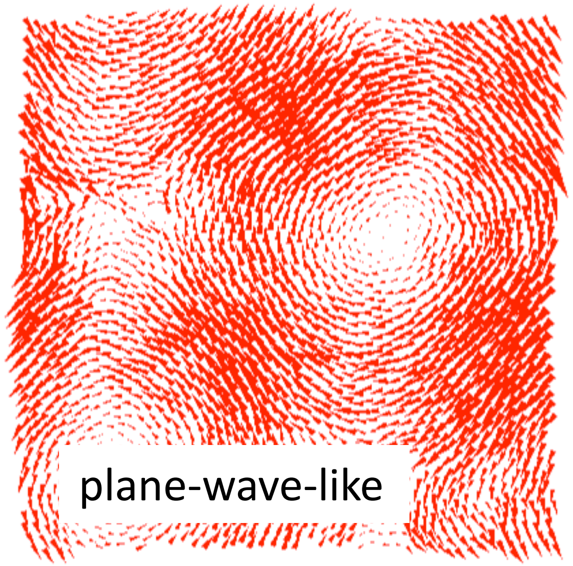
- Crystalline solid
  - Spatially extended phonons are the low-frequency excitations
    - Goldstone’s theorem: broken continuous symmetries generate low-energy long-wavelength excitations
  - Caveat: in crystals with defects, there are resonant modes at the defects

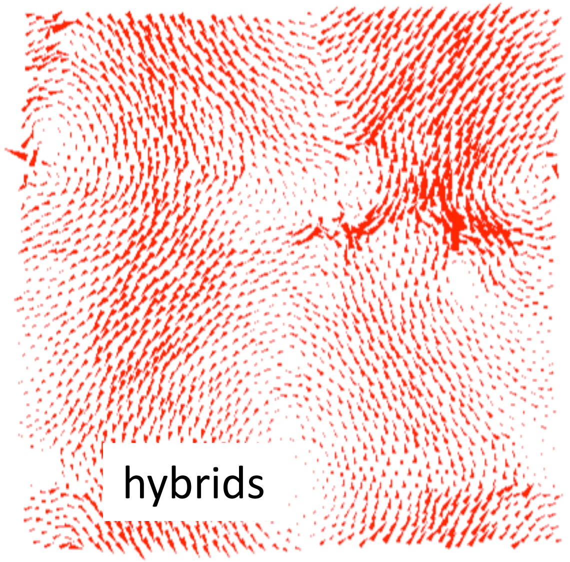
- Disordered solids
  - What are the low-energy excitations?
  - Are they extended or localized?



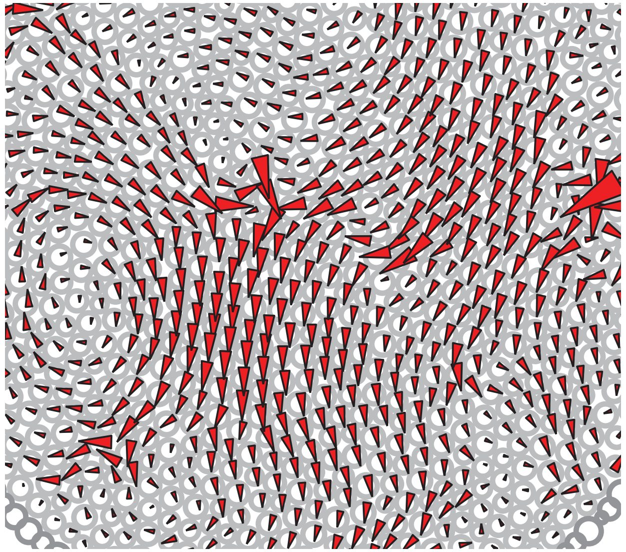
very low frequencies



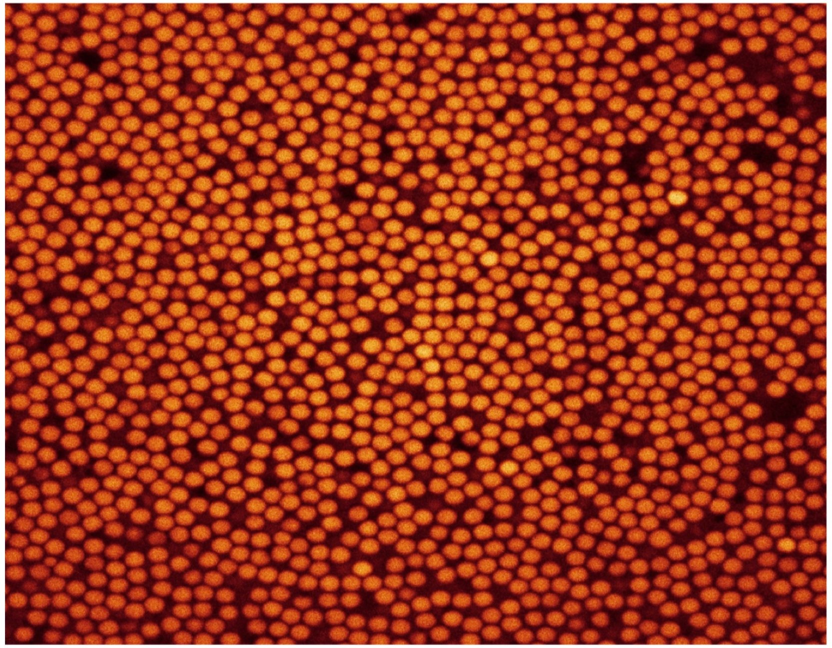




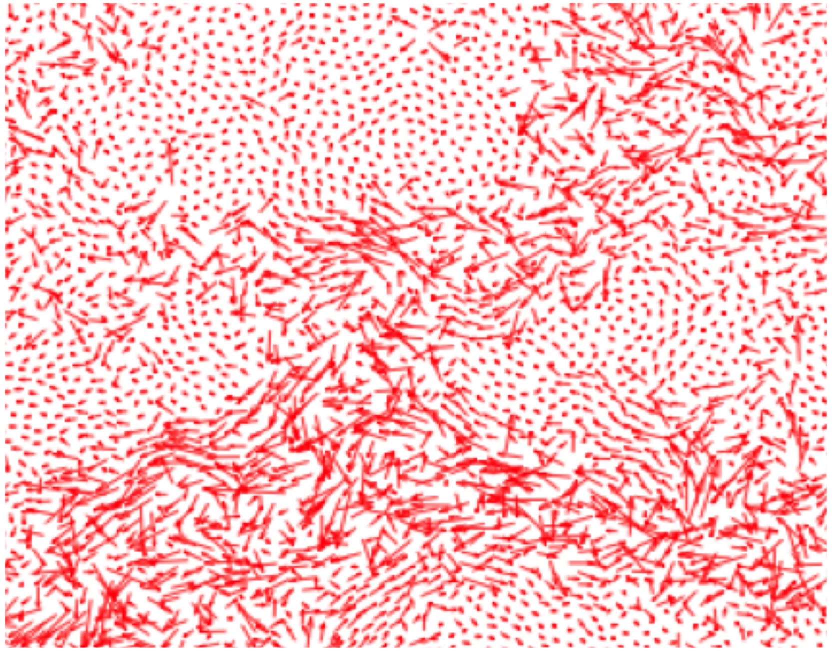
boson peak



Dynamical heterogeneities



A colloidal glass.

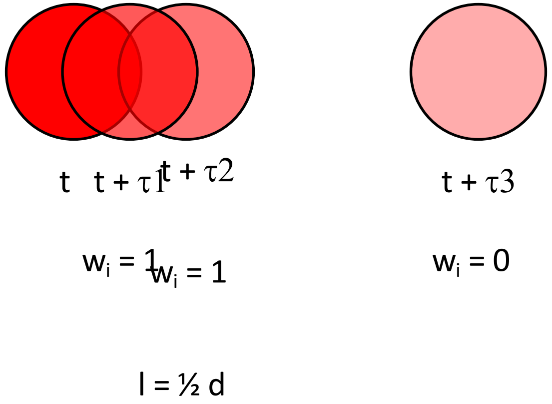


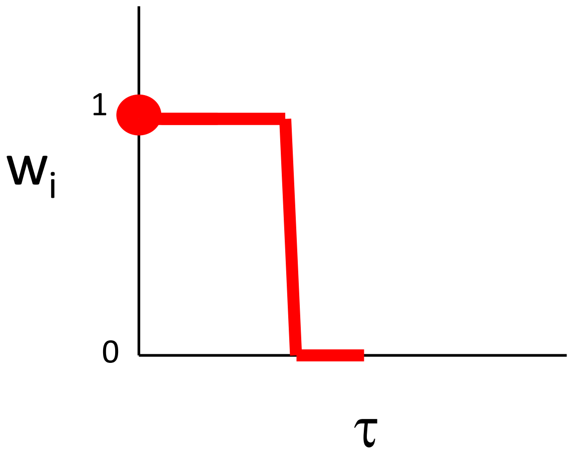
Displacement profile in simulation of a 2-d glass former.
Berthier PRL 2011

Four point correlation functions: captures “swirls” or “dynamical heterogeneities”

$$=< \rho(r', t')\rho(r', t)\rho(r, t')\rho(r, t) >$$

$$Q_t(l, \tau) = \frac{1}{N}\sum_{i=1}^{N} w_i, \quad w_i = \begin{cases} 1, & overlap > l \\ 0, & overlap < l \end{cases}$$

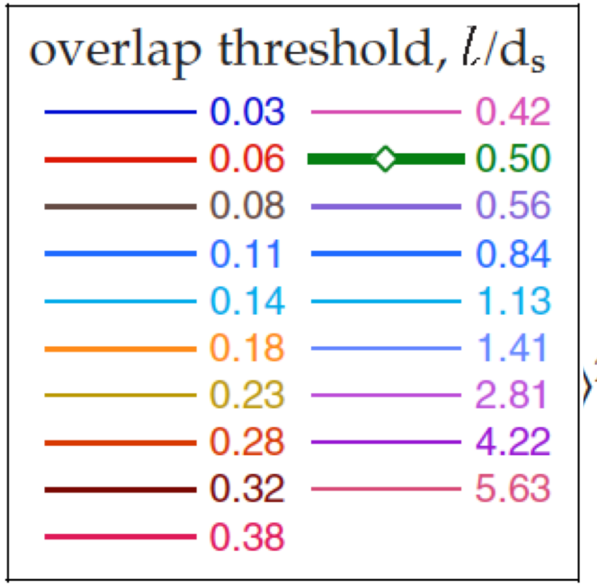




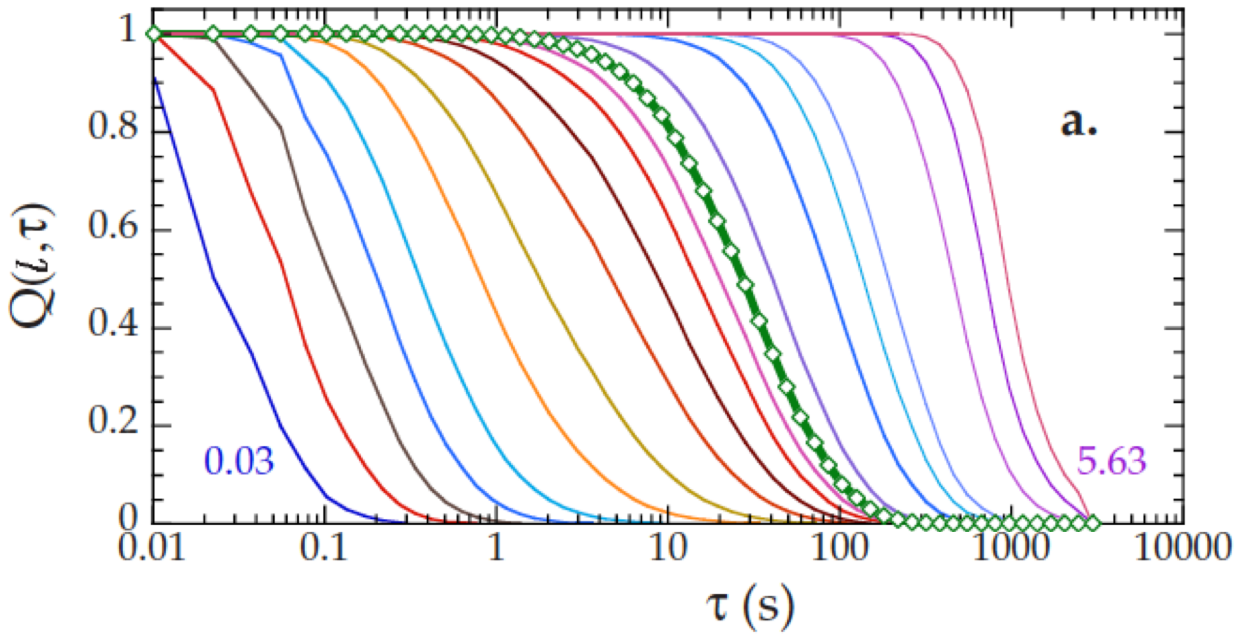
Four point correlation functions

$$Q_t(l,\tau) = \frac{1}{N}\sum_{i=1}^{N} w_i,$$

$$Q(l,\tau) = \langle Q_t(l,\tau)\rangle,$$

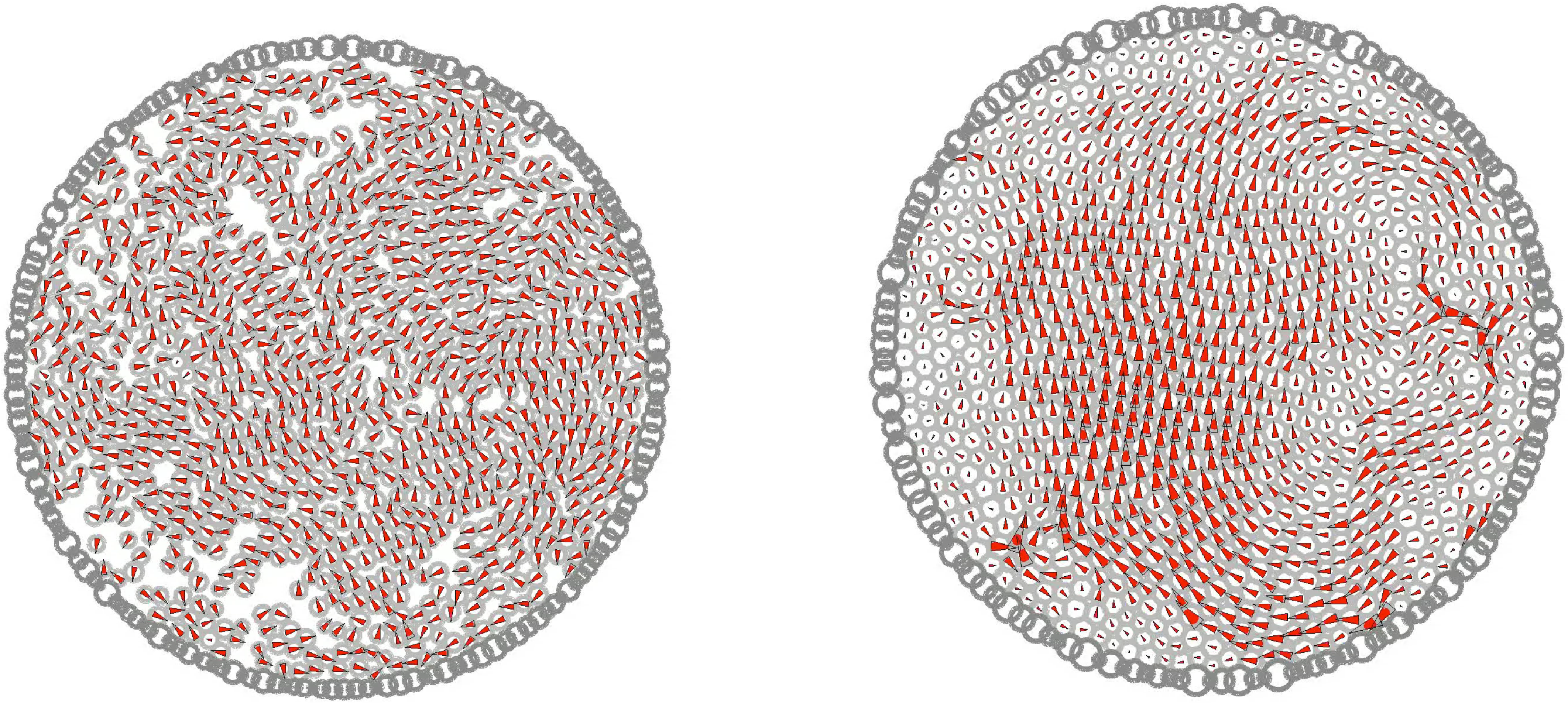


$\chi_4($ $\rangle^2]$



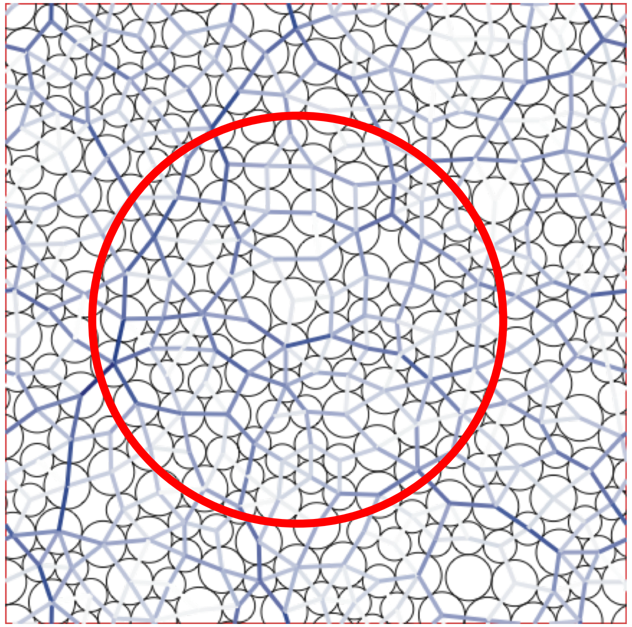
Abate and Durian PRE 2007

Henkes et al PRE 2011



Isostaticity and Diverging Length Scale

M. Wyart, S.R. Nagel, T.A. Witten, EPL **72**, 486 (05)



- For system at ϕ_c, Z=2d
- Removal of one bond makes entire system unstable by adding one soft mode
- This implies diverging length as $\phi \to \phi_c$

For $\phi > \phi_c$, cut bonds at boundary of circle of size L

Count number of soft modes within circle

$$N_s \approx L^{d-1} - (Z - Z_c) L^d$$

Define length scale at which soft modes just appear

$$\ell \approx \frac{1}{Z - Z_c} \approx (\phi - \phi_c)^{-0.5}$$