Problem Set 11
Phys 576, Due THURSDAY, 4/23/2015

1. Impurity Orbits Indium antimonide has $E_g = 0.23$ eV; dielectric constant $\epsilon = 18$; electron effective mass $m_e = 0.015m$. Calculate:
   
   • (a) the donor ionization energy;
   
   • (b) the radius of the ground state orbit.
   
   • (c) at what minimum donor concentration will appreciable overlap effects between the orbits of adjacent impurity atoms occur? This overlap tends to produce an impurity band – a band of energy levels which permit conductivity presumably by a hopping mechanism in which electrons move from one impurity site to a neighboring ionized impurity site.

2. Assuming concentration $n, p$; relaxation times $\tau_e, \tau_h$ and masses $m_e, m_h$, show that the Hall coefficient in the drift velocity approximation (in CGS units) is:

   $$R_H = \frac{1}{ec} \cdot \frac{p - nb^2}{(p + nb)^2};$$

   where $b = \mu_e/\mu_h$ is the mobility ratio. In the derivation, neglect terms of order $B^2$. In SI we drop the c. Hint: In the presence of a longitudinal electric field, find the transverse electric field such that the transverse current vanishes. The algebra may seems tedious, but the result is worth the trouble. Use the result from Problem set 8, problem 1:

   $$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{1 + (\omega_c \tau)^2} \begin{pmatrix} 1 & -\omega_c \tau & 0 \\ \omega_c \tau & 1 & 0 \\ 0 & 0 & 1 + (\omega_c \tau)^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix},$$

   but for two carrier types; neglect $(\omega_c \tau)^2$ in comparison with $\omega_c \tau$.

3. If there is only one carrier type, then in the Drude approximation the motion of charge carriers in electric and magnetic fields does not lead to transverse magnetoresistance. (This was shown in Problem 1 of HW 8.) The result is different with two carrier types. Consider a conductor with a concentration $n$ of electrons of effective mass $m_e$ and relaxation time $\tau_e$; and a concentration $p$ of holes of effective mass $m_h$ and relaxation time $\tau_h$. Treat the limit of very strong magnetic fields, $\omega_c \tau >> 1$. (Again, use the result from HW 8, problem 1).

   • (a) Show in this limit that $\sigma_{yx} = (n - p)ec/B$

   • (b) Show that the Hall field (with $Q \equiv \omega_c \tau$) is given by:

   $$E_y = -(n - p) \left( \frac{n}{Q_e} + \frac{p}{Q_h} \right)^{-1} E_x$$

   which vanishes if $n = p$.

   • (c) Show that the effective conductivity in the $x$-direction is:

   $$\sigma_{eff} = \frac{ec}{B} \left[ \left( \frac{n}{Q_e} + \frac{p}{Q_h} \right) + (n - p)^2 \left( \frac{n}{Q_e} + \frac{p}{Q_h} \right)^{-1} \right].$$

   If $n = p$, $\sigma \propto B^{-2}$. If $n \neq p$, $\sigma$ saturates in strong fields; that is, it approaches a limit independent of $B$ as $B \to \infty$.

4. A semiconductor is doped with $N_D$ donors – at the temperature of interest all donor states are empty. Let the number of electrons in the conduction band be $n_i$ for the intrinsic material at the same temperature.

   • (a) Find an expression for the number $n_0$ of electrons in the conduction band of the doped semiconductor in terms of $N_D$ and $n_i$. Sketch your result in a $\ln(n_0)$ vs. $1/k_bT$ plot.

   • (b) Find the chemical potential $\mu$ for the electrons in the doped semiconductor in terms of $N_D$ and $n_i$. Discuss (and plot) its dependence on $T$ at fixed $N_D$, and then qualitatively discuss how changing $N_D$ alters the plot.